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Static electrical conductivity in weak and moderately non-ideal plasmas

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Abstract. We present a discussion of semiclassical and quantum mechanical RPA treatments of static electrical conductivity in non-ideal plasmas. It is found that the results obtained from both theories agree well with each other in the range of temperatures from 5000 to 50 000 K for electron concentrations between 10^{18} and 10^{20} cm^{-3}. The reported results present a significant improvement on the predictions of the Spitzer formula. Good agreement is also found with available experimental data on non-ideal plasmas. An analytical formula for static conductivity convenient for applications is introduced.

1. Introduction

The static electrical conductivity σ of fully ionized plasma can be expressed in terms of the relaxation time $\tau_e(E)$ in a general integral form as follows:

$$\sigma = -\frac{4e^2}{3m}\int_0^\infty E\rho(E)\tau_e(E)\frac{\mathrm{d}w}{\mathrm{d}E}\,\mathrm{d}E \qquad (1)$$

where $\rho(E)$ is the density of one-electron states in the energy space, $w(E)$ is the equilibrium distribution function, m, e are the mass and the absolute value of charge of the electron. The relaxation time can be defined via the effective frequency $\nu_e(E)$ of electron scattering in plasma, thus

$$\tau_e(E) = \nu_e^{-1}(E). \qquad (2)$$

Equation (1) is usually derived from the moments of the classical Boltzman equation if scattering of electrons on heavy particles is included in consideration but electron–electron scattering is not (see for instance Shkarofsky *et al* (1966)). A special correction is then to be made to account for the neglected electron–electron interaction. Equations (1) and (2) can be taken as a starting point for both quantum-mechanical and semiclassical treatments of the static conductivity in plasmas.

A quantum-mechanical theory for σ has been presented by Adamyan *et al* (1980) and more recently by Djurić *et al* (1991) where frequency of electon scattering $\nu_e(E)$ was calculated in the first Born approximation using Green's function formalism in the random phase approximation (RPA). The treatment in these works was complete in the sense that both electron–electron and electron–ion interactions were included in the RPA calculation of the quantum-mechanical σ^{RPA} using an expansion in terms of polarization operators $\Pi_{a\nu}$ with the ν-summation over the Matsubara frequencies.

The aim of the present paper is to formulate a corresponding semiclassical (SC) theory for static conductivity and compare it with the quantum-mechanical calculations for electron transport in plasmas. In the course of this work, we shall show how the Spitzer theory (Spitzer and Harm 1953, Spitzer 1962) can be obtained from the present model after some simplifications and a special choice of the parameter x_0 below.

The plasma is usually referred to as being ideal if the mean interaction energy of particles is much smaller than their kinetic energy (Anders 1990). The real plasmas where the interaction energy of particles is still significantly smaller than kinetic energy and those where both energies are comparable in magnitude are said to be weakly non-ideal and non-ideal, respectively. Non-ideality can be characterized by the non-ideality parameter Γ,

$$\Gamma = \beta e^2(4\pi N_e/3)^{1/3} \qquad (3)$$

where $\beta = (kT)^{-1}$, N_e and T are free-electron density and plasma temperature. In accordance with the above definitions, $\Gamma \ll 1$ for ideal plasmas and $\Gamma \to 1$ for non-ideal plasmas.

It is well known that the Spitzer theory gives a good account of static conductivity for highly ionized non-degenerate weakly non-ideal plasmas. However it was noted by several authors (Günther *et al* 1976, Günther and Radtke 1984, Kurilenkov and Valuev 1984) that the Spitzer formula overestimates systematically the experimental conductivity of non-ideal plasmas. It is particularly true for singly ionized plasmas with an increased non-ideality Γ. For instance, a significant difference between the value predicted by the Spitzer theory and the experimental data for quasi-stationary dense plasmas ($N_e \approx 10^{18}$ cm^{-3} at $T \approx 10^4$ K) produced in noble-gas-filled flashlamps, has been recently reported by Vitel *et al* (1990).

The thermodynamical domain considered in the present work is wider than that assumed in the original Spitzer theory. Consequently, the results reported below are expected to be valid for an important class of non-ideal plasmas. As we shall see, an additional advantage of the present approach is that it gives results close to the quantum-mechanical calculation in a wide range of plasma conditions. This has also a practical implication because the quantum-mechanical treatment is computationally much more demanding than the semi-classical theory presented here.

2. Semi-classical theory

We shall discuss the case of a two-component quasi-neutral plasma that consists of free electrons and positive ions of charge Ze. We shall take w in equation (1) to be the Fermi–Dirac distribution function,

$$w(E, \mu_{0e}) = 1/(e^{\beta(E-\mu_{0e})} + 1) \qquad (4a)$$

where μ_{0e} is a parameter determined by the normalization condition thus:

$$\int_0^\infty w(E, \mu_{0e})\rho(E)\,dE = N_e \qquad (4b)$$

with

$$\rho(E) = \frac{\sqrt{2}m^{3/2}}{\pi^2\hbar^3}E^{1/2} \qquad (4c)$$

The parameter μ_{0e} in equations (4) is determined in a way similar to that of the RPA theory. Namely, we shall take μ_{0e} to be the chemical potential of an ideal electron gas with the same N_e and T as those in the observed plasma. The ionic component is considered as a homogeneous positive background which ensures the electrical neutrality of the whole system. If the ionic background *is not* homogeneous then effects due to the local structure arise. Earlier Vorob'ev and Khomkin (1977) considered the formation of ionic-atomic clusters in the plasma which may affect the shape of the density function $\rho(E)$. However, these effects are beyond the scope of the present paper since our aim is to establish correspondence between the Spitzer theory and a CS theory based on the RPA, that is between the theories where cluster formation has been neglected. The neglect of these clusters is justified because their contribution in the case of the observed systems (highly ionized and equilibrium gas plasmas) is very small (Khomkin, private communication).

In the RPA approach, ν_e in equation (2) is the effective frequency of the momentum change of a free electron scattered by fluctuations of the internal plasma field that is on ions and electrons. The analogous quantity in the semiclassical theory takes the form

$$\nu_e^{SC}(E) = N_i v Q_{ei}^{tr}(v) + \tfrac{1}{2}N_e\langle|\boldsymbol{v} - \boldsymbol{v}'|Q_{ee}^{tr}(|\boldsymbol{v} - \boldsymbol{v}'|)\rangle \qquad (5)$$

where $N_i = N_e/Z$ is ion density, $v = |\boldsymbol{v}| = (2E/m)^{1/2}$ is the electron velocity in the laboratory frame, and Q_{ei}^{tr} and Q_{ee}^{tr} are transport cross sections for electron–ion and electron–electron collisions, respectively. The bracketed term gives the electron–electron cross section averaged over the electron velocity $\boldsymbol{v}'$ for a given distribution in the plasma. The coefficient $\frac{1}{2}$ in the second term of equation (5) corrects for the double counting of colliding electrons in the gas.

Regarding equation (5), we note that it describes a model of electron scattering in plasma that is closest to the model used in the RPA method. We have justified the use of (5) in equations (1) and (2) by direct comparison of the calculated results with those obtained from the RPA method.

Equation (5) can be written in a convenient form introducing the electron scattering factor χ_{ee} thus:

$$\nu_e^{SC} = \chi_{ee} N_i v Q_{ei}^{tr}(v) \qquad (6a)$$

where

$$\chi_{ee} = 1 + \frac{Z}{2}\frac{\langle|\boldsymbol{v} - \boldsymbol{v}'|Q_{ee}^{tr}(|\boldsymbol{v} - \boldsymbol{v}'|)\rangle}{vQ_{ei}^{tr}(v)} \qquad (6b)$$

so that $\chi_{ee} = 1$ if the electron–electron scattering is neglected.

The transport cross sections Q_{ei}^{tr} and Q_{ee}^{tr} are calculated in the Rutherford approximation with the cut-off impact parameter, thus

$$Q_{ea}^{tr} = \left(\frac{Z_a e^2}{m_{ea}v_{ea}^2}\right)^2 \ln[1 + (r_{ca}m_{ea}v_{ea}^2/Z_a e^2)^2] \qquad (7a)$$

where a = i or e (i and e corresponding to the ion and electron, respectively), $Z_a = Z$ or 1, v_{ea} is the relative velocities of a and e and m_{ae} is the corresponding reduced mass. We take $v_{ei} = v$, $v_{ee} = |\boldsymbol{v} - \boldsymbol{v}'|$ in agreement with (5), $m_{ei} = m$, and define m_{ee} according to

$$m_{ee} = \frac{m^2}{(m+m)\eta} = \frac{m}{2\eta} \qquad (7b)$$

where η is the effective electron mass parameter. The value $\eta = 1$ corresponds to the binary electron–electron collisions on a positive background and $\eta = \frac{1}{2}$ corresponds to a model where the velocity of one electron remains unchanged, that is to the RPA theory. The cut-off radii r_{ce} and r_{ci} will be specified below. Then using equations (2), (5) and (6) we obtain the following expression for the semiclassical relaxation time:

$$\tau_e^{SC}(E) = 1/\nu_e^{SC}(E) = \frac{1}{\chi_{ee}}\frac{(2m)^{1/2}E^{3/2}}{2e^4 Z N_e}\frac{1}{\ln[1+\Lambda_i^2]^{1/2}} \qquad (8)$$

where

$$\Lambda_i = \frac{2\beta E}{p(Z)} \qquad p(Z) = \frac{Z r_L}{r_{ci}} \tag{9}$$

and $r_L = \beta e^2$ is the Landau length.

With the help of (4) and (6) we obtain the following expression for χ_{ee} in equation (8) thus:

$$\chi_{ee} = 1 + \frac{2\pi}{Z}\left(\frac{m}{m_{ee}}\right)^2 \left(\frac{m}{2\pi\hbar}\right)^3 \int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{w(E',\mu_{0e})}{N_e} \times \frac{v^3}{[v_\perp^2 + (v - v_\parallel)^2]^{3/2}} \frac{\ln[1+\Lambda_e^2]^{1/2}}{\ln[1+\Lambda_i^2]^{1/2}} v_\perp \, dv_\perp \, dv_\parallel \tag{10}$$

where

$$E' = \tfrac{1}{2}m(v_\perp^2 + v_\parallel^2) \quad \Lambda_e = \beta m_{ee}[v_\perp^2 + (v - v_\parallel)^2](r_{ce}/r_L) \tag{11}$$

with $v_\perp$ and $v_\parallel$ being the normal and parallel components of $\boldsymbol{v}'$ referring to $\boldsymbol{v}$.

When equation (8) is introduced in (1) and the mean value of the factor $1/\chi_{ee}$ is obtained from the integral, the semiclassical expression for static conductivity in the plasma, σ^{SC}, takes the following form:

$$\sigma^{SC} = \gamma_{ee} A F(\alpha_i, \beta\mu_{0e}) \qquad \gamma_{ee} = \langle 1/\chi_{ee}\rangle . \tag{12}$$

In equation (12),

$$A = \frac{(8/\beta)^{3/2}}{(\pi m)^{1/2} Z e^2} \tag{13}$$

and

$$F(p, \beta\mu_{0e}) = B \int_0^{\infty} \frac{x^3 \exp(x - \beta\mu_{0e}) w^2(x, \beta\mu_{0e}) \, dx}{\ln[1 + \Lambda_i^2(x,p)]^{1/2}} \tag{14}$$

where we have introduced $x = \beta E$. Coefficient B in equation (14) may be expressed in terms of the classical chemical potential. Namely,

$$B = \frac{1}{6} \frac{(m/\beta)^{3/2}}{\sqrt{2}\pi^{3/2}\hbar^3 N_e} = \frac{1}{6}\exp(-\beta\mu_{0e}^{cl}) \tag{15}$$

where μ_{0e}^{cl} is the classical limit of the chemical potential μ_{0e}, that is

$$\mu_{0e}^{cl} = \frac{1}{\beta}\ln\left(\sqrt{2}\pi^{3/2}\hbar^3 N_e/(m/\beta)^{3/2}\right). \tag{16}$$

In the present work, the chemical potential μ_{0e} was obtained numerically, for given N_e and T, from equations (3)-(4). Table 1 presents μ_{0e} and μ_{0e}^{cl} in the range of temperatures T from 3000 to 50 000 K, and in the range of concentrations N_e from 10^{20} to 10^{22} cm^{-3}.

Table 1. The dependence of the parameter $\beta\mu_{0e}$, equations (4) and its classical limit $\beta\mu_{0e}^{cl}$, equation (16), the upper and lower rows, correspondingly, on plasma temperature T and electron density N_e. $\beta = (kT)^{-1}$ and μ_{0e} is chemical potential of the electron component of the plasma.

	$N_e(10^{10}$ cm$^{-3})$				
$T(10^3$ K)	1	5	10	50	100
3	2.027	0.242	0.669	3.903	6.416
	2.071	0.462	0.231	1.841	2.534
5	2.818	1.126	0.330	2.071	3.693
	2.838	1.228	0.535	1.074	1.768
10	3.871	2.232	1.502	0.395	1.440
	3.877	2.268	1.575	0.035	0.728
20	4.915	3.295	2.589	0.877	0.056
	4.917	3.308	2.615	1.005	0.312
30	5.524	3.909	3.209	1.544	0.781
	5.525	3.916	3.223	1.613	0.920
40	5.956	4.343	3.646	2.000	1.261
	5.957	4.347	3.654	2.045	1.352
50	6.291	4.679	3.983	2.347	1.622
	6.292	4.682	3.989	2.380	1.686

The factorized form (12) of the static conductivity is convenient for comparison with the RPA theory as well as with the theory of Spitzer. We note that both σ^{RPA} and σ_{Sp} converge to the limit for the ideal plasma when the non-ideality parameter $\Gamma \to 0$.

We shall now specify the choice of the cut-off radius r_{ca}, a = e or i, in the semiclassical theory. Note that in the RPA theory we deal with the characteristic lengths for a gas constituted of particles of charge $Z_a e$ (a = e, i; $|Z_e| = 1$, $Z_i = Z$) with the corresponding compensating background. Therefore we shall take

$$r_{ca} = \left(\frac{4\pi Z_a^2 e^2}{\partial\mu_{0a}/\partial N_a}\right)^{-1/2} \tag{17}$$

where μ_{0a} is either electronic (a = e) or ionic (a = i) chemical potential and the derivatives are taken at $T =$ constant. For a = e, we have from equations (4) that

$$\frac{\partial\mu_{0e}}{\partial N_e} = \frac{1}{\beta N_e}\left(1 - \frac{1}{\beta^{3/2} N_e}\int_0^{\infty} w^2(x, \beta\mu_{0e})\rho(x)\,dx\right)^{-1}. \tag{18}$$

The ions are treated here as classical particles, for a = i we thus have

$$\frac{\partial\mu_{0i}}{\partial N_i} = \frac{1}{\beta N_i} = \frac{Z}{\beta N_e} \tag{19}$$

From equations (17) and (19) we obtain

$$r_{ci} = r_D^i = (4\pi Z e^2 \beta N_e)^{-1/2}. \tag{20}$$

Equations (19) and (20) directly correspond to a classical gas. It is easy to show that (18) also converges to the correct classical limit. Note that in the classical limit, $\mu_{0e} < 0$ and $x + \beta|\mu_{0e}| \gg 1$, so that (18) is reduced to

$$(\partial\mu_{oe}/\partial N_e)_{cl} = (\beta N_e)^{-1} \quad (21)$$

and

$$r_{ce} = r_D^e = (4\pi e^2 \beta N_e)^{-1/2}. \quad (22)$$

In equations (20) and (22) the characteristic lengths r_D^i and r_D^e are formally equivalent to the Debye radii for an ion and an electron gas with the corresponding compensating homogeneous background at given temperature T.

We have used equation (18) to tabulate the derivative numerically and then obtain r_{ce} from equation (17).

The final expression for the semiclassical static conductivity σ^{SC} is obtained from equation (12) if the function $F(p, \beta\mu_{0e})$, given in the general case by equation (14), is replaced by its classical limit $F_0(p)$ as follows:

$$F_0(p) = \frac{1}{6}\int_0^\infty x^3 e^{-x} \frac{dx}{\ln[1+(2x/p)^2]^{1/2}}$$
$$= \frac{1}{\ln[1+(2x_0/p)^2]^{1/2}} \quad (23)$$

where p is determined by equations (9) and (20) via the chemical potential μ_{0e}, and $x_0 = x_0(p)$ is some 'average' value of x. The expression for σ^{SC} is then given by

$$\sigma^{SC} = \gamma_{ee} A F_0(p) = \frac{A\gamma_{ee}}{\ln[1+(2x_0/p)^2]^{1/2}} \quad (24)$$

where the coefficient A is given by equation (13).

As a result of the transition from F to F_0, σ^{SC} in equation (24) becomes a function of one argument, p, only. This simplifies greatly the comparison between σ^{SC} and Spitzer's conductivity σ_{Sp}. The latter is given by a well-known expression (Spitzer 1962), that is

$$\sigma_{Sp} = A\gamma_{Sp}/\ln(3r_D/Zr_L) \quad (25)$$

where coefficient A is determined, as in equation (24), by equation (13).

We checked that the transition $F \rightarrow F_0$ was justified by direct calculations of x_0 from equation (23) within the entire range of T and N_e covered in table 1. We found that the replacement of F_0 on the left-hand side of equation (23) by the original function $F(p, \beta\mu_{0e})$ results in very small changes in x_0. Within the range $\beta\mu_{0e} \leqslant -1$ in table 1 and outside it with greater T and/or smaller N_e, these changes were less than 1%.

After introducing function F_0, the comparison between σ^{SC} and σ_{Sp} is reduced to a comparison of the electron scattering factors γ_{ee} and γ_{Sp} and of the arguments of the corresponding logarithms, in a region of weak and ideal plasmas, $\Gamma \rightarrow 1$.

Let us start with the electron scattering factor γ_{ee} in equation (24). Numerical calculations have shown that for each given N_e and Z, γ_{ee} has the following properties as a function of temperature

Table 2. Coefficient $\gamma_{ee} = \langle\chi_{ee}^{-1}\rangle$ determined by equations (6) and (10) as a function of plasma temperature T, for the case of electron density $N_e = 10^{18}$ cm^{-3} and $Z = 1$, 2 and 4.

	Z		
$T(10^3$ K)	1	2	4
10	0.630	0.697	0.687
20	0.604	0.697	0.754
30	0.590	0.694	0.766
40	0.582	0.691	0.771
50	0.576	0.689	0.774
60	0.572	0.688	0.776
70	0.569	0.687	0.777
80	0.567	0.686	0.778
90	0.565	0.686	0.779
100	0.563	0.685	0.780
$\gamma_{Sp}(Z)$	0.582	0.683	0.785

(i) Its values always lie in a narrow interval.

(ii) It has a maximum in a low temperature domain outside the range of table 1.

(iii) It decreases monotonically with temperature in such a way that $\gamma_{ee} = \gamma_{Sp}$ for some value of T within the range of table 1.

The changes in γ_{ee} within the full domain of T have been found rather small, less than 10% of Spitzer's value γ_{Sp}. This is demonstrated in table 2 where values of both factors are compared at $T > 10^4$ K and $N_e = 10^{18}$ cm^{-3} for $Z = 1$, 2 and 4. The values of γ_{ee} given in table 2 have been computed with the effective mass parameter $\eta = \frac{1}{2}$ in equation (7b). It has been found that, with this choice of η (corresponding to the RPA method), the best agreement between σ^{SC} and σ_{Sp} is achieved. This choice of η ensures also a minimum departure of γ_{ee} from γ_{Sp}. Correspondingly, we shall use

$$\gamma_{ee} = \gamma_{Sp}(Z). \quad (26)$$

Now we shall turn to the logarithmic term in the right-hand side of equation (23). Let us consider the case of $\Gamma \ll 1$. Then the unit in the log argument can be neglected in comparison with the second term there. We can expand the resulting expression as follows:

$$F_0(p) = \frac{1}{\ln(1/p)}(1 - Q + Q^2 - \ldots) \qquad Q = \frac{\ln(2x_0)}{\ln(1/p)}. \quad (27)$$

It follows from (27) that the asymptotic value of F_0 as $\Gamma \rightarrow 0$, does not depend on a particular numerical choice of x_0. For non-ideal plasmas, however, the choice of x_0 *does* affect the value of F_0.

We note that the Spitzer logarithm in equation (25) is obtained from (27) if we take E as being the mean thermal value, $E = \frac{3}{2}kT$, that is $x_0 = \frac{3}{2}$. This value may not be the best choice for the 'average' x because, as can easily be seen, the integrand in equation (23) peaks at $x = 3$ rather than at $x = \frac{3}{2}$. A more consistent way of dealing with this integral is to evaluate it exactly,

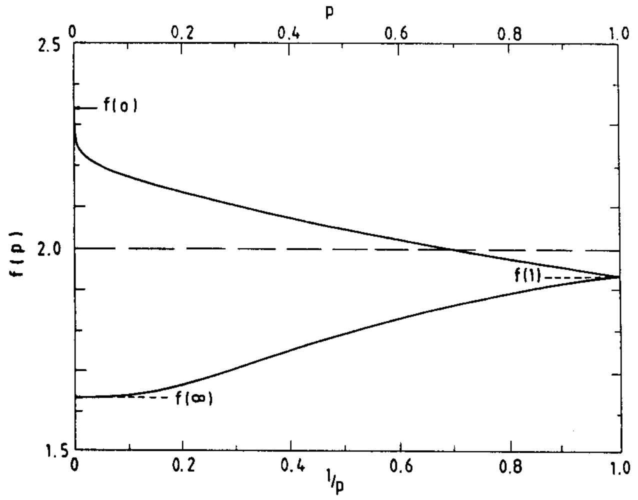

Figure 1. The function $f(p)$, equation (28). Upper curve: $f(p)$ for $p \leqslant 1$; lower curve: $f(p)$ for $p \geqslant 1$ given as a function of $1/p$. Special values of $f(p)$: $f(0) = 2.3412$, $f(1) = 1.9365$ and $f(\infty) = 1.6330$.

for given p, and replace the p-independent x_0 by a p-dependent parameter, according to

$$x_0 = \frac{3}{2} f(p) \qquad p = \frac{Z r_L}{r_{ci}}. \tag{28}$$

For $f = 1$, $x_0 = \frac{3}{2}$ in accordance with the Spitzer convention. Therefore the function $f(p)$ characterizes the departure of $x_0(p)$, for a given value of p, from Spitzer's choice of x_0. The parameter p used in the present paper (see also equation (9)) coincides with the parameter for a non-ideal plasma which is often denoted in the literature as γ.

The function $f(p)$ in (28) has been computed in the entire range $0 \leqslant p < \infty$ in our previous paper (Mihajlov *et al* 1991). We have tabulated $f(p)$ and displayed results in figure 1. For small p, this function is given by the following for $p =$ 0, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1} we have, respectively, $f(p) =$ 2.341, 2.291, 2.281, 2.264, 2.236, 2.172.

In the range $0.1 \leqslant p \leqslant 1$, $f(p)$ is accurately represented by a simple linear function:

$$f(p) = 2.198 - 0.262p. \tag{29}$$

As figure 1 shows, $f(p)$ is close to 2 over a wide range of p and never reaches 1. In other words, the 'average' value of x in integral (23) is indeed much closer to 3 (where the integrand attains a maximum) rather than to Spitzer's value of 1.5.

Using equation (26) for γ_{ee} and the definition of x_0 given by equation (28), we obtain the following final expression for σ^{SC}:

$$\sigma^{SC} = \frac{8(2kT)^{3/2}}{(\pi m)^{1/2} Z e^2} \frac{\gamma_{Sp}(Z)}{\ln[1 + (2x_0/p)^2]^{1/2}} \tag{30}$$

where $x_0 = x_0(p)$, p is given by equation (28) and r_{ci} is given by (20). Equation (30) ensures automatically the asymptotic equivalence (when $\Gamma \to 0$) of the semiclassical conductivity σ^{SC} and Spitzer's conductivity σ_{Sp}. This is an important result considering that σ^{SC} must play the role of a classical analogue of σ^{RPA} which, in turn, has been shown to be asymptotically equivalent to σ_{Sp} (Djurić *et al* 1991). The coefficient $\gamma_{Sp}(Z)$ in equation (30) was tabulated by Spitzer (1962) for selected values of charge Z. Its values for Z = 1, 2 and 4 are given in table 2. Therefore, equation (30) determines completely our semiclassical model of conductivity.

3. Comparison with the quantum mechanical theory

The quantum mechanical and semiclassical conductivities, σ^{RPA} and σ^{SC}, respectively, were compared for the particular case of a hydrogen-like plasma with $Z = 1$. For the quantum-mechanical model, the effective frequency $\nu_e(E) = \nu_e^{RPA}(E)$ in equations (1) and (2) was obtained using the following algorithm:

$$\nu^{RPA}(E) = \frac{e^2 m N_e kT}{(2mE)^{3/2}} \int_0^{q(E)} q\,dq \sum_\nu \frac{\epsilon_\nu(q) - 1}{\epsilon_\nu^3(q)}$$

$$q(E) = \frac{(8mE)^{1/2}}{h} \tag{31}$$

$$\epsilon_\nu(q) = 1 + \frac{4\pi e^2}{q^2} \sum_{a,\nu} Z_a^2 \Pi_{a\nu}(q)$$

where q is the momentum of the electron, $\epsilon_\nu(q)$ is the static dielectric function, $\Pi_{a\nu}(q)$ is the polarization operator of species of sort a, and ν-summation is over the Matsubara frequencies. Functions $\epsilon_\nu(q)$ and $\Pi_{a\nu}(q)$ are defined in Djurić *et al* (1991). The results are found to be dependent on the slow convergence of the Matsubara series in ν in equation (31). Even for weakly non-ideal non-degenerate plasmas ($N_e = 10^{18}$ and $T = 5 \times 10^4$ K) some 100 terms in the series (31) are required to ensure computational accuracy at a level of 1%. The calculations become even more protracted in the case of a many-component plasma. Therefore the semiclassical model developed in the present paper is a practically useful alternative since it requires only limited computational effort.

The semiclassical conductivity σ^{SC} was computed from equations (28)–(30) with γ_{Sp} taken to be 0.582 (for $Z = 1$).

A comparison of the two sets of static conductivities is presented in figure 2 where the conductivity curves are shown in the temperature range between 5×10^3 and 5×10^4 K for electron densities $N_e = 10^n$, $n = 18$, 19 and 20.

It is easy to see that, for high temperatures when the non-ideality of plasmas decreases, all curves for given N_e and T converge to the same asymptotic limit. However, as T decreases and the parameter of non-ideality Γ becomes larger, the curves diverge from each other. The deviation becomes most apparent at temperatures $T \leqslant 2 \times 10^4$ K. For $T > 10^4$ K, the semiclassical model gives results that are very close to those from the RPA calculation.

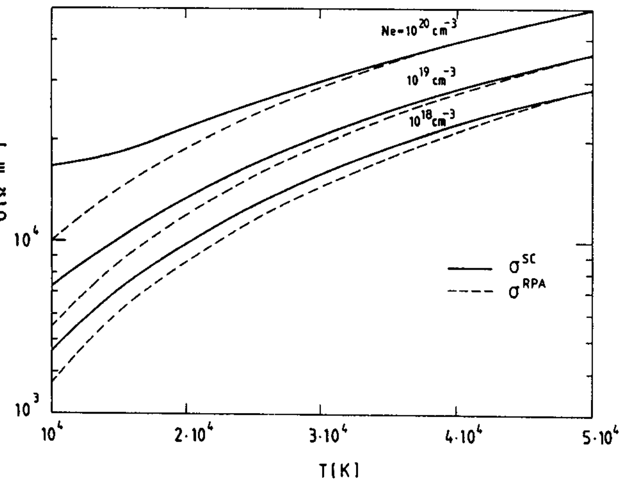

Figure 2. Comparison of σ^{SC} and σ^{RPA} in a selected domain of T and N_e.

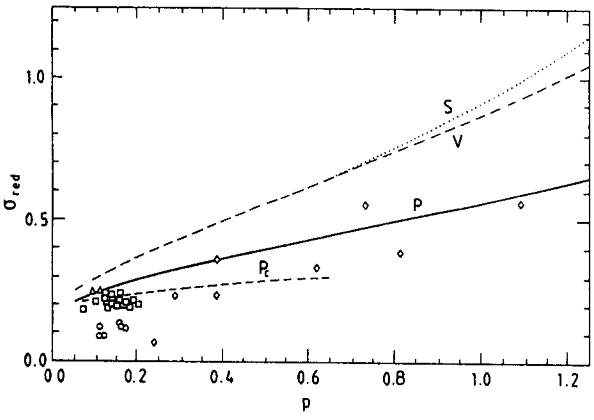

Figure 3. Comparison of theoretical and experimental values of $\sigma_{red} = \sigma(A\gamma_{Sp})^{-1}$. Theory: S, dotted curve, Spitzer theory; V, broken curve, modification of Vitel *et al* (1990); P, full curve, present theory for the Debye cut-off parameter, $q_c = 1$. Also shown: P_c, broken curve, present theory for a non-Debye cut-off parameter, with $q_c \neq 1$ determined in section 4. Experiment: (◇) Ivanov *et al* (1976); (Δ) Günther and Radtke (1984); (□) Vitel *et al* (1990); and ○ Benage *et al* (1990).

4. Discussion

In figure 3 we compare the available experimental data on the static electrical conductivity of the singly ionized (hydrogen-like) plasmas with the theoretical curves for $Z = 1$ drawn in the reduced form, $\sigma^{SC}_{red} = 1/\ln[1 + (3f(p)/p)^2]^{1/2}$ determined by equations (28)–(30). It is seen that the experimental points lie closer to σ^{SC}_{red} rather than to the corresponding theoretical curves of Spitzer (1962) and Vitel *et al* (1990).

Vitel *et al* (1990) found that the line profiles obtained in their work indicated that the plasma became less collisional when non-ideality increased. At the same time they observed, as the present figure 3 shows, that the measured static electrical conductivity was significantly lower than that according to Spitzer's theory. In order to resolve this contradiction they suggested that there was an additional scattering by the oscillating microfields in such dense plasmas which led to the reduced conductivity. It is a matter of interest to point out that this additional mechanism in non-ideal plasmas is expected to be more pronounced in regions where the gradients of temperature and density are high (Kurilenkov and Valuev 1984). However, this condition was not satisfied in the experiments of Vitel *et al* (1990), who carried out the determination of the current in the axial region where the gradients were small. The present work suggests that there is indeed no compelling reason for assuming strong effects of such additional scattering in the experimentally tested conditions.

Another way of accounting for the non-ideality of plasmas would be to consider quantum scattering at the cut-off Coulomb or Debye potentials, both depending on a cut-off parameter r_c (Günther and Radtke 1984). However, in the range of experimental conditions treated in the present paper, transport cross sections and, therefore, static conductivity depend very little on the particular model of the scattering centre.

Finally, we shall discuss one particular modification of the presented SC theory. It was earlier suggested by Günther *et al* (1976) that, for non-ideal dense plasmas, the cut-off parameter r_{ci} should be different from the Debye radius r^i_D given by equation (20). This requirement may be taken into account by replacing r_{ci} in equation (30) by $q_c r^i_D$ where the scaling parameter q_c depends on the number of particles, n_D, inside the Debye sphere (that is inside a sphere of radius r^i_D). For instance, the model considered by Kaklyugin and Norman (1973) gives, for $Z = 1$,

$$q_c = 1 + \frac{1}{n_D} + \frac{2}{5}\frac{\ln n_D}{n_D} \qquad n_D \gtrsim 1. \qquad (32)$$

Equations (28) and (30) above may be considered as a particular case ($q_c = 1$) of the scaled theory. We note that the modified value of the cut-off parameter, for which p in equation (28) has to be replaced by p/q_c, changes slightly the relation between the experimental conditions (N_e and T) and the numerical value of the parameter. As figure 3 shows, the rescaled curve P_c for $\sigma^{SC}_{red}(p)$ gives an even better fit to the experiment than the curve P does for the Debye cut-off (q_c=1).

The present theory can be readily extended, in a simple form, to the general case of many-component plasmas by introducing the effective ionic charge $\tilde{Z}$ to replace Z, thus:

$$\tilde{Z} = N_e^{-1} \sum Z^2 N_Z \qquad N_e = \sum Z N_Z \qquad (33)$$

where summation is carried out over all sorts of ions in the plasma.

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