

The New Screening Characteristics of Strongly Non-Ideal and Dusty Plasmas. Part 2: Two-Component Systems¹

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Abstract—A new model method for describing of the electrostatic screening in two-component systems (electron-ion plasmas, dusty plasmas, electrolytes, etc) is developed. The method is applicable to the systems of higher non-ideality degree. The expressions for all the screening parameters introduced in the previous paper (Part 1) of this work, as well as for an additional parameter characteristic for multi-component systems, are obtained. All these parameters are presented in a simple analytic form suitable for operative laboratory usage, especially for theoretical interpretation of experimental data.

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1. INTRODUCTION

In the previous paper [1], here Part 1, the aims of this research were already described, as well as the stimuli for its starting. Accordingly to these aims the new model method of describing of the electrostatic screening in electron-ion plasmas and other two-component systems (e.g. dusty plasmas and some electrolytes) which relies on the basic model (a1) – (a3) is presented in this part. This method is free of non-physical properties of Debye-Hückel's (DH) method and posses the positive features (b1) and (b2) described in Section 1 of Part 1, together with the basic model.

2. THEORY ASSUMPTIONS

2.1. Screening Model

A stationary homogeneous two-component system S_{in} is taken here as the initial model of some real physical objects. We will assume that S_{in} is constituted by a mix of two gases: one of positive charged ions (of only one kind), and other of electrons. It is assumed that these gases there are in the equilibrium states with temperatures T_i and $T_e > T_i$, and mean local particle density N_i and N_e . All the particles are treated as point objects with the charge $Z_e e$ in the case of electron, and $Z_i e$ in the case of ion, where $Z_e = -1$, $Z_i = 1, 2, \dots$, and e is the modulus of the electron charge. Let us note that in this paper the electron charge will be also denoted by $-e$. It is understood that the parameters $Z_{e,i}$ and N_e , is satisfy the local quasi-neutrality condition

$$Z_i e N_i - e N_e = 0 \quad (1)$$

as well as that N_e and T_e allow the non-relativistic treatment of the electron component.

In accordance with the properties (a1) and (a2), the screening of a charged particles in the system S_{in} will be modeled in the corresponding accessory systems each of which differs from S_{in} in that, besides the two described components, it also contains a fixed probe particle with charge $Z_p e$ in the origin of the chosen reference frame (point O). Here we will study two cases: the ion case (i), when $Z_p = Z_i$, and the electron case (e), when $Z_p = Z_e = -1$, when the probe particle represents one of the particles of the system S_{in} . In accordance with this, we will denote here the corresponding accessory system with $S_a^{(i)}$ or $S_a^{(e)}$. This system will be characterized by: the ion and electron densities $n_e^{(i,e)}(r)$ and $n_i^{(i,e)}(r)$, the mean local charge density

$$\rho^{(i,e)}(r) = Z_i e n_i^{(i,e)} - e n_e^{(i,e)}(r), \quad (2)$$

and the mean electrostatic potential $\Phi^{(i,e)}(r)$, where $r = |\mathbf{r}|$ and \mathbf{r} is the radius-vector of the observed point. It is assumed the satisfying of the boundary conditions

$$\lim_{r \rightarrow \infty} n_i^{(i,e)}(r) = N_i, \quad \lim_{r \rightarrow \infty} n_e^{(i,e)}(r) = N_e, \quad (3)$$

and the condition of neutrality of the systems $S_a^{(i,e)}$ as wholeness

$$Z_{i,e} e + \int_0^{\infty} \rho^{(i,e)}(r) \times 4\pi r^2 dr = 0. \quad (4)$$

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Then, we will take into account that the $\Phi^{(i,e)}(r)$ and $\rho^{(i,e)}(r)$ have to satisfy Poisson's equation

$$\nabla^2 \Phi^{(i,e)} = -4\pi[Z_{i,e}e\delta(\mathbf{r}) + \rho^{(i,e)}(r)], \quad (5)$$

where $\delta(\mathbf{r})$ is three-dimensional delta function [2]. From the same reason as in Part 1, this equation applies in the whole region $r > 0$. It is assumed the satisfying the boundary conditions

$$\lim_{r \rightarrow \infty} \Phi^{(i,e)}(r) = 0, \quad (6)$$

$$|\Phi^{(i,e)}| < \infty, \quad \varphi^{(i,e)} \equiv \lim_{r \rightarrow 0} [\Phi^{(i,e)}(r) - Z_{i,e}e/r]. \quad (7)$$

Since $\varphi^{(i,e)}$ is the mean electrostatic potential in the point O , the quantity

$$U^{(i,e)} = Z_{i,e}e\varphi^{(i,e)} \quad (8)$$

is the potential energy $U^{(i,e)}$ of the probe particle. In an usual way $U^{(i)}$ and $U^{(e)}$ are treated as approaches to the mean potential energies of the ion and electron in the system S_{in} .

In accordance with the properties (a2) and (a3) the conditions of thermodynamical equilibrium of the ion component in the case (i), as well as the electron component in the case (e), will be taken in the form

$$\begin{aligned} \mu_{i,e}(n_{i,e}^{(i,e)}(r), T_{i,e}) + Z_{i,e}e\Phi^{(i,e)}(r) \\ = \mu_{i,e}(N_{i,e}, T_{i,e}), \end{aligned} \quad (9)$$

where $\mu_i(n_i^{(i)}(r), T_i)$ and $\mu_e(n_e^{(e)}(r), T_e)$ are the chemical potentials of the ideal ion and electron gases, which can depend of the corresponding particle spins, considered on the distance r from the point O . On the base of the considerations from Part 1 one should keep in mind that the equations (9) are applicable only in the regions

$$r \geq r_{s;i,e}, \quad r_{s;i,e} \equiv \left(\frac{3}{4\pi N_{i,e}} \right)^{1/3}, \quad (10)$$

where $r_{s;i}$ and $r_{s;e}$ are the corresponding Wigner-Seitz's radii. In the used procedure the equation (9) is taken in the linearized form

$$\begin{aligned} n_{i,e}^{(i,e)}(r) - N_{i,e} &= -\frac{Z_{i,e}e}{\partial\mu_{i,e}/\partial N_{i,e}} \Phi^{(i,e)}(r), \\ \partial\mu_{i,e}/\partial N_{i,e} &\equiv \left[\frac{\partial\mu_{i,e}(n, T_{i,e})}{\partial n} \right]_{n=N_{i,e}} \end{aligned} \quad (11)$$

but under the condition

$$\frac{|n_{i,e}^{(i,e)}(r) - N_{i,e}|}{N_{i,e}} \ll 1. \quad (12)$$

It is important that the conditions (10) and (12) are compatible in all considered cases.

Since we take the single-component systems considered in Part 1, as a boundary case of two-component systems (when it is spread one of their components), we will require that the ion density $n_i^{(i)}$ in the case (i) and

the electron density $n_e^{(e)}$ in the case (e) satisfy the equation

$$\int_0^\infty [N_{i,e} - n_{i,e}^{(i,e)}(r)] \times 4\pi r^2 dr = 1, \quad (13)$$

which is analogous of the equation (21) from Part 1. From Eqs. (4) and (13) it follows that the electron density $n_e^{(i)}$ in the case (i) and ion density $n_i^{(e)}$ in the case (e) have to satisfy another equation

$$\int_0^\infty [n_{e,i}^{(i,e)}(r) - N_{e,i}] \times 4\pi r^2 dr = 0. \quad (14)$$

Let us emphasize that the relations (13) and (14) can be obtained on the base of the interpretation of the systems $S_a^{(i,e)}$ which is given in Appendix D. In further considerations is used the fact that simultaneous satisfying of the conditions (13) and (14) automatically provide the satisfying of neutrality condition (4).

2.2. The Critical Analysis of DH Method

The procedure of obtaining of DH solutions is described in Appendix A. The Figure 1 shows the behavior of the particle densities $n_{D;i}^{(i)}(r)$ and $n_{D;e}^{(i)}(r)$, and charge density $\rho_D^{(i)}(r)$. This figure illustrates that the behavior of DH densities of the free particles with the same charge as the probe particle is qualitatively same as in the case which is considered in the Part 1. Consequently, the procedure of the elimination of the non-physical properties of $n_{D;i}^{(i)}(r)$ in the case (i), and $n_{D;e}^{(e)}(r)$ in the case (e) will be similar to the procedure which is described in Part 1.

The main disadvantage of DH method consists in the monotonous increasing of DH densities of the free particles, which the charge is opposite to the charge of the probe particle, with the decreasing of r in the whole region $r < \infty$. This fact is illustrated by the behavior of $n_{D;e}^{(i)}(r)$ in Fig. 1. Because of such a behavior DH solutions principally can not satisfy the conditions (14). Namely, from results of Appendix A it follows that in DH case the left side of those conditions is not equal to zero, but it is proportional to Z_i in the case (i), and $1/Z_i$ in the case (e). The consequence of this fact is the principal impossibility to treat the probe particle as a represent of a particle in the system S_{in} . For an example, in the case of completely classical electron-ion plasma with $Z_i = 1$ and $T_i = T_e$ from the non-satisfying of the conditions (14) it follows that the mean number of electrons per ion should be $3/2$, instead of 1.

The described disadvantage is a consequence of two facts: that in DH method the first step is determining of

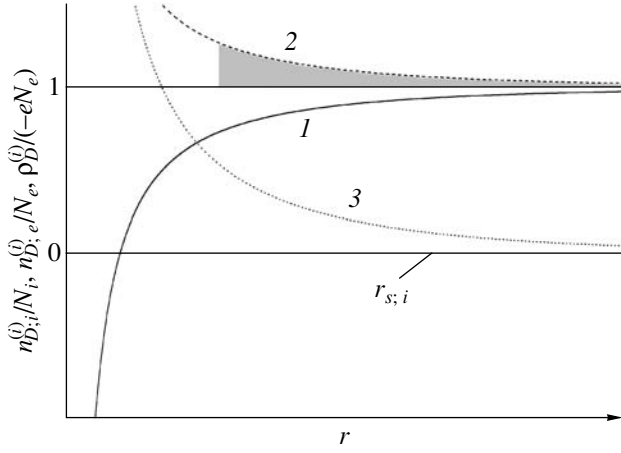


Fig. 1. The reduced DH densities $n_{D;i}^{(i)}/N_i$ (1), $n_{D;e}^{(i)}/N_e$ (2) and $\rho_D^{(i)}/(-eN_e)$ (3) in the case $Z_i = 1$, $T_e = T_i$ и $\kappa_D r_{s;i} = 1$, where κ_D is Debye's screening constant given by (57). Shading emphasize the non-physical deviation of $n_{D;e}^{(i)}$ from the asymptotic value N_e .

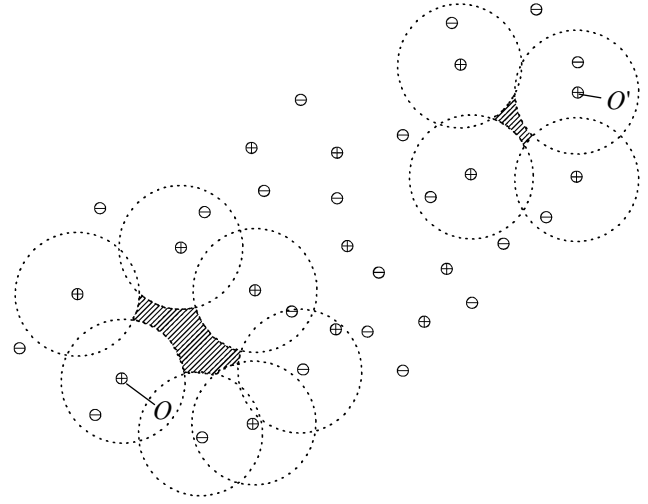


Fig. 2. The schematic picture of ion-electron plasma. Shaded areas represent the regions of r without of probe particle's and all ion's self-shells. With O and O' the places of the probe particle in the case (i) and one of ions are denoted, and with the dotted lines probe particle and ions self spheres are shown.

electrostatic potential $\Phi_D^{(i,e)}(r)$ in the whole space, and the equations (11) are used together with the equations (56) in Appendix A. Because of that within DH method the electron and ion components are “smeared” in the space simultaneously and independently in both (i) and (e) cases.

2.3. What One Should Do in Order to Avoid Non-Physicality of DH Method?

In accordance with above mentioned, our main task is finding of such a procedure within the basic model (see Section 1 in Part 1), which would be alternative one to DH procedure. It assumes that sought procedure has to provide the possibility of determining of the solutions $n_i^{(i)}(r)$ and $n_e^{(i)}(r)$ in the same way as it was described in Part 1, and the solutions $n_i^{(i)}(r)$ and $n_e^{(i)}(r)$ without using the first equations (56) Appendix A. Also, the satisfying of the conditions (13) and (14) is assumed.

3. THE PRESENTED METHOD: THE ELECTRON AND ION DENSITIES

3.1. The Case (i)

The solution $n_i^{(i)}(r)$. In order to solve our task we start from such a picture of the system $S_a^{(i)}$ where electrons are treated in the electrostatic field of the probe particle and all ions (treated in a classic way) which are distributed in discrete points, as it is illustrated by Fig. 2. On the base of this picture, the procedure of the

expressing of $n_e^{(i)}(r)$ in the region $r > r_{s;i}$ through $n_i^{(i)}(r)$ is developed in the first part of Appendix B. The results of this procedure can be presented in the form

$$n_e^{(i)}(r) = N_e(1 - \alpha) + \alpha Z_i n_i^{(i)}(r), \quad (15)$$

$$r_{s,i} < r < \infty;$$

where the parameter α will be determined in further text. This relation, together with Eq. (2), makes possible to represent the charge density $\rho^{(i)}(r)$ in the form

$$\rho^{(i)}(r) = Z_i e(1 - \alpha)[n_i^{(i)}(r) - N_i], \quad r_{s,i} < r < \infty, \quad (16)$$

and to use for determination of $n_i^{(i)}(r)$ the procedure which is described in details in Sections 3 and 6 of Part 1. As first, by means, Eqs. (11) and (16) it is obtained the equation of Volterra's type, namely

$$\rho^{(i)}(r) = \kappa_i^2 \int_r^\infty \rho^{(i)}(r') \left(\frac{1}{r} - \frac{1}{r'} \right) r'^2 dr', \quad (17)$$

where

$$\kappa_i \equiv \frac{1}{r_{\kappa;i}} = \kappa_{0;i}(1 - \alpha)^{1/2}, \quad \kappa_{0;i} = \left[\frac{4\pi(Z_i e)^2}{\partial \mu_i / \partial N_i} \right]^{1/2}. \quad (18)$$

One can see that in the two-component case the ion screening constant κ_i and the corresponding characteristic length $r_{\kappa;i}$ depend on the parameter $(1 - \alpha)^{1/2}$.

Accordingly to Part 1, the solution of the equation (17) in the general case can be taken in the form $\rho^{(i)}(r) = c_i Z_i e \exp(-\kappa_i r)/r$. From here and Eq. (17) it follows that ion density $n_i^{(i)}(r)$ in the region $r_{s;i} < r < \infty$ is given by

the relation $n_i^{(i)}(r) = N_e - c_i \exp(-\kappa_i r)/r$. Than, taking $c_i = r_{0;i} \exp(\kappa_i r_{0;i}) N_e$ and applying to this relation the extrapolation procedure from Part 1, we obtain the sought expression for $n_i^{(i)}(r)$ in the whole space

$$n_i^{(i)}(r) = \begin{cases} N_i - N_i r_{0;i} \exp(\kappa_i r_{0;i}) \frac{\exp(-\kappa_i r)}{r}, & r_{0;i} < r, \\ 0, & r \leq r_{0;i}, \end{cases} \quad (19)$$

where the new characteristics length $r_{0;i}$ has to be determined from the condition (13). Using the results of Part 1 we can present the radius $r_{0;i}$ in two equivalent forms

$$r_{0;i} = \gamma_s(x_i) r_{s;i}, \quad r_{0;i} = \gamma_\kappa(x_i) r_{\kappa;i}, \quad (20)$$

$$x_i = \kappa_i r_{s;i}, \quad (21)$$

where the coefficients $\gamma_s(x)$ and $\gamma_\kappa(x)$ are given by Eq. (28) from Part 1, and $r_{s;i}$ —by Eq. (10).

The solution $n_e^{(i)}(r)$. In order to determine $n_e^{(i)}(r)$ in the probe particle self-sphere ($0 < r < r_{s;i}$), we will take into account that in the system S_{in} the mean number of electrons in the sphere with volume $1/N_i$, which is centered at some ion (the ion self-sphere), is larger than the mean number of electrons in every fixed sphere with the same volume, i.e. $N_e(1/N_i) = Z_i$, because of the additional electrons whose presence is caused by presence of other ions. In accordance with this we will find $n_e^{(i)}(r)$ in the probe particle self-sphere in the system $S_a^{(i)}$ in the form

$$n_e^{(i)}(r) = n_{e;s}(r) + n_{e;ion}(r), \quad 0 < r < r_{s;i}, \quad (22)$$

where the member $n_{e;s}(r)$ satisfies the condition

$$\int_0^{r_{s;i}} n_{e;s}(r) \times 4\pi r^2 dr = Z_i \quad (23)$$

corresponding to that this member describes the distribution of Z_i electrons which there are in the probe particle self-sphere independently of the presence of ions, while the member $n_{e;ion}(r)$ describes the distribution of the mentioned additional electrons. Let us emphasize that such a treatment of the difference $[n_e^{(i)}(r) - n_{e;s}(r)] \equiv n_{e;ion}(r)$ is caused by the procedure of the obtaining the member $n_{e;s}(r)$ which is described in the second part of Appendix B.

In order to establish the connections between the members in Eqs. (15) and (22), we will assume that the form of the equation (15), which transforms at $r = \infty$ to equality $N_e = N_e(1 - \alpha) + N_e\alpha$, reflects the existence in the system S_{in} of the spatial correlation between electrons and ions. It means exceptionally following: in every fixed volume V in average there are $VN_e(1 - \alpha)$

electrons independently of presence of ions, and $VN_e\alpha$ electrons whose presence is caused by the presence of ions. The fact that the presence of ions in some volume causes the presence of additional electrons itself has already been used above, while here it is assumed that just the parameter α in Eq. (15) represents the quantitative characteristic of the mentioned electron-ion correlation in the system $S_a^{(i)}$. Because of that the first of the mentioned connections is given by the relation

$$n_{e;ion}(r) = \alpha Z_i n_i^{(i)}(r), \quad 0 < r \leq r_{s;i}, \quad (24)$$

which corresponds to the way of obtaining of the ion density, and provides the satisfaction of three conditions: that the member $n_{e;ion}(r)$ in (22) has to be close to $\alpha Z_i n_i^{(i)}(r)$ at least in one part of the region $r < r_{s;i}$; in whole this region the behavior of $r < r_{s;i}$ has to reflect the behavior of $n_{e;ion}(r)$; the ratio of the mean numbers of additional electrons and ions in the probe particle self-sphere has to be equal to correlation coefficient, i.e. α under adopted conditions. Another of the mentioned connections is the condition

$$n_{e;s}(r_{s;i}) = N_e(1 - \alpha), \quad (25)$$

which provides needed continuity of $n_e^{(i)}(r)$ at $r = r_{s;i}$. The physical sense of this condition is discussed in the further text.

The procedure of determination of the member $n_{e;s}(r)$ is described in the second part of Appendix B. The first results of this procedure are the equations for differences $n_{e;s}(r) - n_{e;s}(r_{s;i})$ and $\Phi_s(r_{s;i}) - \Phi_s(r)$ in Appendix B, which provide the obtaining of the equation for direct determination of $n_{e;s}(r)$, namely

$$\begin{aligned} n_{e;s}(r) - \frac{\kappa_{0;e}^2 Z_i}{4\pi r} - \kappa_{0;e}^2 \int_0^r n_{e;s}(r') \left(\frac{1}{r'} - \frac{1}{r} \right) r'^2 dr' \\ = n_{e;s}(r_{st}) - \frac{\kappa_{0;e}^2 Z_i}{4\pi r_{st}} - \kappa_{0;e}^2 \int_0^{r_{st}} n_{e;s}(r') \left(\frac{1}{r'} - \frac{1}{r_{st}} \right) r'^2 dr', \end{aligned} \quad (26)$$

where the screening parameter $\kappa_{0;e}$ is given by

$$\kappa_{0;e} = \left(\frac{4\pi e^2}{\partial \mu_e / \partial N_e} \right)^{\frac{1}{2}}. \quad (27)$$

The next results of the used procedure are the presentation $n_s(r)$ as the sum of two exponents multiplied by r which gives the member $n_{e;s}(r)$ in the form

$$\begin{aligned} n_{e;s}(r) = N_e \left[a \frac{r_{s;i}}{r} \exp\left(-x_s \frac{r}{r_{s;i}}\right) \right. \\ \left. + b \frac{r_{s;i}}{r} \exp\left(x_s \frac{r}{r_{s;i}}\right) \right], \quad 0 < r \leq r_{s;i}, \end{aligned} \quad (28)$$

where the coefficients a and b have to satisfy the condition

$$a + b = \frac{Z_i \kappa_{0;e}^2}{4\pi r_{s;i} N_e} \equiv \frac{(\kappa_{0;e} r_{s;i})^2}{3} \quad (\text{see Appendix B}),$$

which provides that Eq. (28) really represents a solution of the equation (26), as well as the condition $a \exp(-\kappa_{0;e} r_{s;i}) + b \exp(\kappa_{0;e} r_{s;i}) = 1 - \alpha$, which provides the satisfying of the condition (25). Under these conditions a and b are obtained in the form

$$a = \frac{1 - \alpha - \frac{1}{3} x_s^2 \exp(x_s)}{\exp(-x_s) - \exp(x_s)}, \quad (29)$$

$$b = -\frac{1 - \alpha - \frac{1}{3} x_s^2 \exp(-x_s)}{\exp(-x_s) - \exp(x_s)},$$

$$x_s = \kappa_{0;e} r_{s;i}. \quad (30)$$

Finally, the last free parameter, i.e. the correlation coefficient α , is determined from the condition (23) and is given by the expression

$$\alpha = 1 - \frac{\frac{2}{3} x_s^3}{(1 + x_s) \exp(-x_s) - (1 - x_s) \exp(x_s)}. \quad (31)$$

From here it follows that α satisfies the conditions

$$\begin{aligned} 0 < \alpha < 1, \quad 0 < x_s < \infty; \\ \lim_{x_s \rightarrow 0} \alpha = 0; \quad \lim_{x_s \rightarrow \infty} \alpha = 1; \end{aligned} \quad (32)$$

which make possible the treatment of α as the correlation coefficient in the whole region $0 < x_s < \infty$. Finally, by means of Eqs. (15), (22), (24) and (25) the electron density $n_e^{(i)}(r)$ can be presented in the form

$$n_e^{(i)}(r) = \begin{cases} N_e(1 - \alpha) + \alpha Z_i n_i^{(i)}(r), & r_{s;i} < r < \infty, \\ n_{e;s}(r) + \alpha Z_i n_i^{(i)}(r), & 0 < r < r_{s;i}, \end{cases} \quad (33)$$

where $n_i^{(i)}(r)$ and $n_{e;s}(r)$ are given by Eqs. (19) and (28)–(30), which provide $n_e^{(i)}(r)$ satisfies the condition Eq. (14), while a_i and b_i are given by (29) and α —by (31).

It can be shown that $n_{e;s}(r)$, given by (28)–(31), in the region $r \leq r_s$ monotonically increases with decreasing of r and satisfies the equality

$$\left[\frac{dn_{e;s}(r)}{dr} \right]_{r=r_{s;i}} = 0, \quad 0 < x_s < \infty. \quad (34)$$

Apart of that, these facts provide smoothness of the electron density $n_e^{(i)}(r)$ in the point $r = r_{s;i}$ and give the clear physical sense to the condition (25) within the developed method. Namely, we have that just the value $n_{e;s}(r_{s;i})$ in this condition determines the value of the effective density of the non-correlated part of the elec-

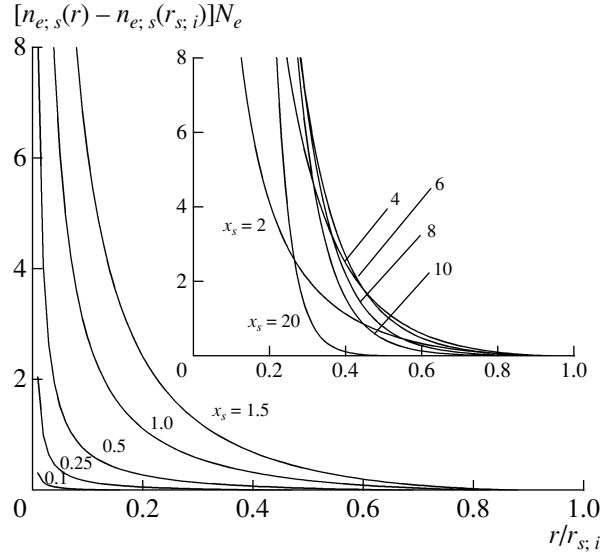


Fig. 3. The behavior of the ratio $[n_{e;s}(r) - n_{e;s}(r_{s;i})]/N_e$, for $0.1 \leq x_s = \kappa_{0;e} r_{s;i} \leq 20$.

tron component, i.e. $N_e(1 - \alpha)$, inside and outside of the probe particle self-sphere.

Finally, the property (34) of $n_{e;s}(r)$ causes such a behavior of the difference $[n_{e;s}(r) - n_{e;s}(r_{s;i})]$, which is illustrated by Fig. 3. This figure shows that the deviation of that difference from zero can be practically neglected within the layer $0.75r_{s;i} \leq r \leq r_{s;i}$ for any $x_s > 0$.

This provides the extrapolation of the $n_i^{(i)}(r)$ from the region $r > r_{s;i}$ to the part of region $r < r_{s;i}$ which makes about 60% of the probe particle self-sphere can be performed in the same way as in Part 1, which means that the deviation of $[n_{e;s}(r) - n_{e;s}(r_{s;i})]$ from zero can be neglected. It is clear that such a behavior of $n_{e;sc}(r)$, which could not be expected in advance, additionally justifies the applied procedure.

3.2. The Case (e)

The solution $n_e^{(e)}(r)$. In this case treatment of the light (electron) and hard (ion) components appears as the main problem. The way of its solving and the procedure of the expressing $n_i^{(e)}(r)$ through $n_e^{(e)}(r)$ in the region $r_{s;e} < r < \infty$ are described in first part of Appendix C. The results of this procedure is the expression for α in Appendix C which can be presented in the form

$$n_i^{(e)}(r) = N_i(1 - \alpha) + \frac{\alpha}{Z_i} n_e^{(e)}(r), \quad r_{s;e} < r < \infty. \quad (35)$$

This relation, together with Eq. (2), makes possible to express the charge density $\rho^{(e)}(r)$ in the form: $\rho^{(e)}(r) = -e[n_e^{(e)}(r) - N_e](1 - \alpha)$, and repeat word-for-word the procedure from Section 3.1. As result we obtain the

needed expression for the electron density in the whole space

$$n_e^{(e)}(r) = \begin{cases} N_e - N_e r_{0;e} \exp(\kappa_e r_{0;e}) \frac{\exp(-\kappa_e r)}{r}, & r_{0;e} < r < \infty, \\ 0, & 0 < r \leq r_{0;e}, \end{cases} \quad (36)$$

$$\kappa_e \equiv \frac{1}{r_{\kappa,e}} = \kappa_{0;e} (1 - \alpha)^{\frac{1}{2}}, \quad (37)$$

where $\kappa_{0;e}$ is defined by (27), and the electron screening constant κ_e and the corresponding characteristic length $r_{\kappa,e}$ depend on the parameter $(1 - \alpha)^{1/2}$. The radius $r_{0;e}$, similarly to the case (i), can be presented in two equivalent forms

$$r_{0;e} = \gamma_s(x_e) r_{s;e}, \quad r_{0;e} = \gamma_\kappa(x_e) r_{\kappa;e}, \quad (38)$$

$$x_e = \kappa_e r_{s;e}, \quad (39)$$

where $\gamma_s(x)$ and $\gamma_\kappa(x)$ are given by relations (28) from Part 1, and $r_{s;e}$ —by Eq. (10).

The solution $n_i^{(e)}(r)$. Repeating the procedure from Section 3.1 we will find the ion density $n_i^{(e)}(r)$ inside the probe particle self-sphere, $0 < r \leq r_{s;e}$, in the form

$$n_i^{(e)}(r) = n_{i;s}(r) + n_{i;el}(r), \quad n_{i;el}(r) = \frac{\alpha}{Z_i} n_e^{(e)}(r), \quad (40)$$

where the member $n_{i;s}(r)$ satisfies the conditions

$$\int_0^{r_{s;e}} n_{i;s}(r) \times 4\pi r^2 dr = \frac{1}{Z_i}, \quad (41)$$

$$n_{i;s}(r_{s;e}) = N_i(1 - \alpha), \quad (42)$$

which play similar role as the conditions (23) and (25) in the case (i).

The way of determination of $n_{i;s}(r)$ is described in second part Appendix C, and as the result $n_{i;s}(r)$ is presented as a superposition of exponents multiplied by r^{-1} . After determination of the coefficients in the superposition from the conditions (41) and (42), the member $n_{i;s}(r)$ can be presented in the form

$$n_{i;s}(r) = N_i \left[a \frac{r_{s;e}}{r} \exp\left(-x_s \frac{r}{r_{s;e}}\right) + b \frac{r_{s;e}}{r} \exp\left(x_s \frac{r}{r_{s;e}}\right) \right], \quad 0 < r \leq r_{s;e}, \quad (43)$$

where a and b are given by Eq. (29) and x_s —by Eq. (30). Finally, from Eqs. (35), (40) and (43) it follows the expression for $n_i^{(e)}(r)$ in the hole space

$$n_i^{(e)}(r) = \begin{cases} N_i(1 - \alpha) + \frac{\alpha}{Z_i} n_e^{(e)}(r), & r_{s;e} < r < \infty, \\ n_{i;s}(r) + \frac{\alpha}{Z_i} n_e^{(e)}(r), & 0 < r \leq r_{s;e}, \end{cases} \quad (44)$$

where $n_e^{(e)}(r)$ and $n_{i;s}(r)$ are given by Eqs. (36)–(39) and (43), which provide that $n_i^{(e)}(r)$ satisfies the condition (14), and α is given by Eq. (31).

It can be shown that the member $n_{i;s}(r)$ monotonously increases in the region $r < r_{s;e}$ with the decreasing of r and satisfies the equality

$$\left[\frac{dn_{i;s}(r)}{dr} \right]_{r=r_{s;e}} = 0, \quad 0 < x_s < \infty, \quad (45)$$

which provides smoothness of $n_i^{(e)}(r)$ in the point $r = r_{s;e}$. Consequently, it guarantees that the ion density $n_i^{(e)}(r)$ in the case (e) has the similar properties as the electron density $n_e^{(i)}(r)$ in the case (i).

4. THE SOLUTIONS $\rho^{(i,e)}(r)$ AND $\Phi^{(i,e)}(r)$, AND THE PROBE PARTICLES POTENTIAL ENERGIES $U^{(i,e)}$

By means Eqs. (2), (19), (33), (36) and (44) the charge densities $\rho^{(i,e)}(r)$ can be presented in the form

$$\rho^{(i,e)}(r) = \rho_\alpha^{(i,e)}(r) + \rho_{\alpha;s}^{(i,e)}(r), \quad (46)$$

$$\rho_\alpha^{(i,e)}(r) = Z_{e,i} e (1 - \alpha) [N_{i,e} - n_{i,e}^{(i,e)}(r)], \quad (47)$$

$$0 < r < \infty,$$

$$\rho_{\alpha;s}^{(i,e)}(r) = Z_{e,i} e [n_{e,i;s}^{(i,e)}(r) - N_{e,i} (1 - \alpha)], \quad (48)$$

$$0 < r < r_{s;i,e},$$

where $n_{i,e}^{(i,e)}(r)$ and $n_{e,i;s}^{(i,e)}(r)$ are given by Eqs. (19) and (36), and Eqs. (28) and (43), respectively. One can see that the expression (47) has the same structure as the corresponding expression from Part 1 and becomes identical to it for $\alpha = 0$, while (48) describes the contribution of the quantity $n_{e,i;s}^{(i,e)}(r) - N_{e,i} (1 - \alpha)$, which characterizes just two-component systems.

In accordance with the structure of the expressions for $\rho^{(i,e)}(r)$ we will find the electrostatic potentials

$\Phi^{(i,e)}(r)$, as well as the potentials $\varphi^{(i,e)}$ defined by Eqs. (7), in the form

$$\begin{aligned}\Phi^{(i,e)}(r) &= \Phi_{\alpha}^{(i,e)}(r) + \Phi_{\alpha;s}^{(i,e)}(r), \\ \varphi^{(i,e)} &= \varphi_{\alpha}^{(i,e)} + \varphi_{\alpha;s}^{(i,e)},\end{aligned}\quad (49)$$

where the first and second members describe the contributions of the members $\rho_{\alpha}^{(i,e)}(r)$ and $\rho_{\alpha;s}^{(i,e)}(r)$ in Eq. (46). It can be shown that by means Eqs. (47) and (48), as well as Eqs. (51)–(53) from Part 1, the members $\Phi_{\alpha}^{(i,e)}(r)$ and $\Phi_{\alpha;s}^{(i,e)}(r)$ can be presented in the form

$$\Phi_{\alpha}^{(i,e)}(r) = \frac{Z_{i,e}e(1-\alpha)}{r} \begin{cases} \chi(x_{i,e}) \exp(-\kappa_{i,e}r), & r_{0;i,e} < r < \infty, \\ 1 + \frac{\Phi_{\alpha}^{(i,e)}}{Z_{i,e}e(1-\alpha)} + \frac{1}{2} \left(\frac{r}{r_{s;i,e}} \right)^3, & 0 < r \leq r_{0;i,e}, \end{cases} \quad (50)$$

$$\begin{aligned}\Phi_{\alpha;s}^{(i,e)}(r) &= \frac{Z_{i,e}e}{r} \left[\alpha + \frac{\Phi_{\alpha;s}^{(i,e)} r}{Z_{i,e}e} + 3 \frac{a \exp\left(-x_s \frac{r}{r_{s;i,e}}\right) + b \exp\left(x_s \frac{r}{r_{s;i,e}}\right) + (a-b)x_s \frac{r}{r_{s;i,e}}}{x_s^2} - 1 \right] \\ &\quad - \frac{Z_{i,e}e(1-\alpha)}{r} \frac{1}{2} \left(\frac{r}{r_{s;i,e}} \right)^3, \quad 0 < r \leq r_{s;i,e},\end{aligned}\quad (51)$$

where x_i and x_e are defined by Eqs. (21) and (39), x_s —by Eq. (30), the function $\chi(x)$ —by Eq. (30) from Part 1. The quantities $\Phi_{\alpha}^{(i,e)}$ and $\Phi_{\alpha;s}^{(i,e)}$; s are obtained by means Eqs. (47) and (48), as well as Eq. (52) from Part 1, and presented in the form

$$\Phi_{\alpha}^{(i,e)} = -Z_{i,e}e(1-\alpha) \frac{3r_{0;i,e}}{2r_{s;i,e}^3} \left(r_{0;i,e} + \frac{2}{\kappa} \right), \quad (52)$$

$$\begin{aligned}\Phi_{\alpha;s}^{(i,e)} &= -\frac{3Z_{i,e}e}{2r_{s;i,e}} \\ &\times \left[2 \frac{a(1 - e^{-x_s}) + b(e^{x_s} - 1)}{x_s} - (1-\alpha) \right],\end{aligned}\quad (53)$$

where the coefficients a and b are given by Eqs. (29), and the radii $r_{0;i,e}$ —by Eqs. (20), (21), (38) and (39), as well as Eq. (28) from Part 1.

In accordance with Eqs. (8), (49), (52) and (53) it is appropriate to find the potential energies $U^{(i,e)}$ of the probe particles in the cases (i) and (e) in the form

$$U^{(i,e)} = U_{\alpha}^{(i,e)} + U_{\alpha;s}^{(i,e)} \equiv Z_{i,e}e\varphi_{\alpha}^{(i,e)} + Z_{i,e}e\varphi_{\alpha;s}^{(i,e)}, \quad (54)$$

where the quantities $\varphi_{\alpha}^{(i,e)}$ and $\varphi_{\alpha;s}^{(i,e)}$ are given by Eqs. (52), (53). It can be shown that, because of the structure of Eq. (52), the member $U_{\alpha}^{(i,e)}$ can be taken as: $U_{\alpha}^{(i,e)} = (1-\alpha)U$, with U given by one of two equivalent expressions (32) and (33) from Part 1, where the parameters Z , x , r_{κ} and r_s are replaced by $Z_{i,e}$, $x_{i,e}$, $r_{\kappa;i,e}$ and $r_{s;i,e}$. The Figure 4 illustrates the behavior of the ratio of the probe particle potential energy, determined by means to Eqs. (52), (53) and (54), and the corre-

sponding DH energy, defined in Appendix A, in the case of the classical plasma with $Z_i = 1$ and $T_e = T_i$. This figure shows that in two-component case the DH region, i.e. the region where this ratio is close to unity, principally does not exist.

5. RESULTS AND DISCUSSIONS

The expressions (19)–(33) and (36)–(50) show that *the obtained solutions satisfy all conditions from Section 2, are free of all non-physical properties of DH solutions, and possesses the positive properties (b1) and (b2), noted in Section 1 of Part 1.* The behavior of electron, ion and the charge density is illustrated by Fig. 5. The way of their obtaining provides the possibility of their application, independently of the treatment (classical or quantum mechanical) of electron and ion components of the considered systems. Apart of that, the structure of these expressions makes possible their application not only to electron-ion plasmas, but also to ion-ion two-component systems (e.g. some electrolytes).

Since Eqs. (28), (33), (43) and (44) show that the solutions $n_{e,i}^{(i,e)}(r)$ are singular in the point $r = 0$, it is useful to note that the existence of singularities in model solutions is fully acceptable, if it has not other non-physical consequences. Such solutions are well known in physics: it is enough to mention, for example, Thomas-Fermi's models of electron shells of heavy atoms [3, 4] (see also [5]), which used in plasma research till now (see e.g. [6]).

In the case of two-component system are obtained the parameters $r_{0;i,e}$, $\gamma_s(x_{i,e})$ and $\gamma_{\kappa}(x_{i,e})$, given by Eqs. (20), (21), (38) and (39), analogous to that ones

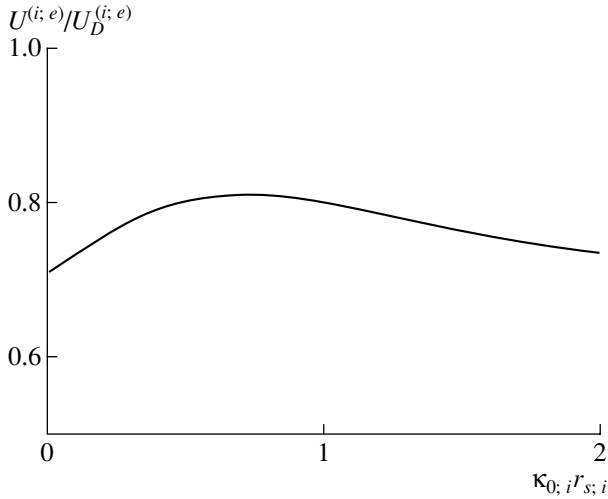


Fig. 4. The behavior of the ratio $U^{(i,e)}/U_D^{(i,e)}$ as a function of the parameter $\kappa_{0,i} r_{s,i}$ in the case of completely classical plasma with $Z_i = 1$ and $T_i = T_e$, where $\kappa_{0,i}$ is defined by Eq. (18).

from Part 1. In accordance with Eq. (28) from Part 1 the parameters $r_{0,i}$ and $r_{0,e}$ satisfy the conditions

$$\begin{aligned} 0 < r_{0,i,e} < r_{s,i,e}, \quad 0 < x_{i,e} < \infty; \\ \lim_{x_{i,e} \rightarrow 0} r_{0,i,e} = 0; \quad \lim_{x_{i,e} \rightarrow \infty} r_{0,i,e} = r_{s,i,e}; \end{aligned} \quad (55)$$

where $r_{s,i,e}$ is given by Eq. (10), and x_i and x_e —by Eqs. (21) and (39). These conditions make possible the treatment of $r_{0,i}$ and $r_{0,e}$ as the radii of the spheres centered on the probe particles, which are classically forbidden for the free charged particles from their neighborhoods, and $\gamma_s(x_{i,e})$ and $\gamma_\kappa(x_{i,e})$ —some kind of non-ideality parameters (see Part 1). From (55) it follows also that Eqs. (19), (33), (36) and (50) for electron and ion densities are applicable to the two-component systems with any non-ideality degree.

Also, in this paper is obtained the quantity $\alpha(x_s)$, defined by Eqs. (30) and (31), which has the sense of the coefficient of electron-ion correlation. Let us note that two simple approximative expressions for $\alpha(x_s)$, which serve very well in wide region of x_s , will be given in Part 3.

One of the most important results of this papers is establishing of the fact that in two-component plasmas ion and electron components have to be described exceptionally by means of screening constants κ_i and κ_e , and the corresponding screening radii $r_{\kappa,i}$ and $r_{\kappa,e}$; e which are introduced in this paper. This means that *Debye-Hückel's screening constant κ_D and the radius r_D do not appear in the theory and, consequently, principally do not have the physical sense.* It is confirmed by Fig. 4 where the probe particle potential energies obtained here are compared with the corresponding DH values. Finally, comparison of the expressions (49),

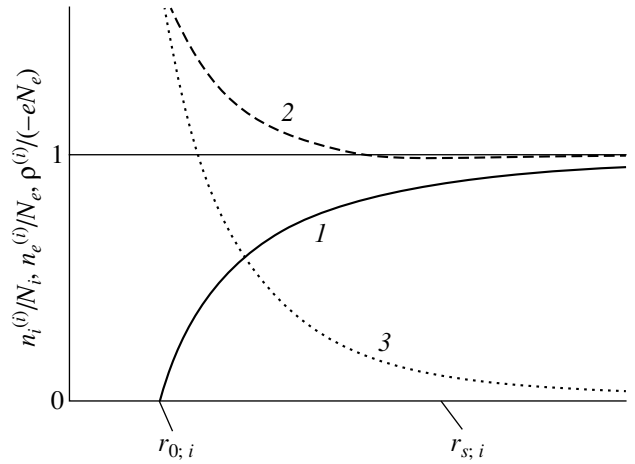


Fig. 5. The behavior of reduced densities $n_i^{(i)}(r)/N_i$ (1), $n_e^{(i)}(r)/N_e$ (2), and $\rho^{(i)}(r)/(-eN_e)$ (3) in the case $Z_i = 1$, $T_i = T_e$, $\kappa_{D,i} r_{s,i} = \kappa_{D,e} r_{s,i} = 1$, where $\kappa_{0,i}$ is defined by (18).

(50) and (51) for the potentials $\Phi^{(i,e)}(r)$ with the expression (58) for DH solutions $\Phi_D^{(i,e)}(r)$ in Appendix A shows that in two-component case the principal difference between these solutions there is not only inside the probe particle self-spheres, but also in the rest of space.

Namely, out of these self-spheres $\Phi_D^{(i,e)}(r) \sim \exp(-\kappa_D r)$, where D is defined in Appendix A, while the solution $\Phi^{(i,e)}(r) \sim \exp(-\kappa_{i,e} r)$, where κ_i and κ_e are given by Eq. (18), (27) and (37). This fact justifies the usage in [7–10] the constants which are close to κ_i and κ_e instead of DH constants.

CONCLUSIONS

The analysis of non-physical properties of Debye-Hückel's method was continued in this paper in the case of two-component systems and it was found the procedure for their elimination within the same basic model. Owing to that it is developed a new model method for describing of the inner-plasma electrostatic screening in two-component systems (electron-ion and dusty plasmas, some electrolytes, etc.), which is free of the mentioned non-physical properties and could be applied for the systems with higher non-ideality degree.

It was demonstrated that all new screening parameters, determined in the previous paper for the single-component systems (one characteristic length and two non-ideality parameters), have the analogous parameters which characterize each component in two-component systems. Besides, it was introduced into consideration a new parameter (the coefficient of the electron-ion correlation) which characterizes just two-component systems.

APPENDICES

APPENDIX A

DH Solutions

In DH method it is assumed that for determination of $n_e^{(i)}$ and $n_i^{(e)}$, apart of Eqs. (11), it can be applied the equations

$$\begin{aligned} n_{e,i}^{(i,e)}(r) - N_{e,i} &= -\frac{Z_{e,i}e}{\partial\mu_{e,i}/\partial N_{e,i}}\Phi^{(i,e)}(r), \\ \partial\mu_{e,i}/\partial N_{e,i} &\equiv \left[\frac{\partial\mu_{e,i}(n, T_{e,i})}{\partial n} \right]_{n=N_{e,i}}, \end{aligned} \quad (56)$$

which represent the linearized form of the equations: $\mu_{e,i}(n_{e,i}^{(i,e)}(r), T_{e,i}) + Z_{e,i}e\Phi^{(i,e)}(r) = \mu_{e,i}(N_{e,i}, T_{e,i})$. Within DH method equations (11) and (56) apply together in order to express the charge densities in the Poisson's equations (5) through the electrostatic potential in the whole region $0 < r < \infty$. That way, in accordance with Eqs. (1)–(3) and (5) one obtains Helmholtz's equation

$$\begin{aligned} \nabla^2\Phi^{(i,e)}(r) &= \kappa_D^2\Phi^{(i,e)}(r), \\ \kappa_D &\equiv \frac{1}{r_D} = (\kappa_{0,i}^2 + \kappa_{0,e}^2)^{\frac{1}{2}}, \end{aligned} \quad (57)$$

where κ_D is DH screening constant, and $\kappa_{0,i}$ and $\kappa_{0,e}$ are the partial screening constants defined by Eqs. (18) and (27).

DH electrostatic potential $\Phi_D^{(i,e)}(r)$ represent the solutions of equation (57), which is obtained in the whole space under the boundary conditions (6) and (7). Then, by means of Eqs. (11) and (56) DH solutions for the charge and particle densities are obtained. All these solutions are given by the expressions

$$\Phi_D^{(i,e)}(r) = \frac{Z_{i,e}e}{r}\exp(-\kappa_D r), \quad (58)$$

$$\rho_D^{(i,e)}(r) = -\frac{Z_{i,e}e\kappa_D^2\exp(-\kappa_D r)}{4\pi r},$$

$$n_{D,i}^{(i)}(r) = N_i - \frac{\kappa_{0,i}^2\exp(-\kappa_D r)}{4\pi r}, \quad (59)$$

$$n_{D,e}^{(e)}(r) = N_e - \frac{\kappa_{0,e}^2\exp(-\kappa_D r)}{4\pi r},$$

$$n_{D,i}^{(i)}(r) = N_e + \frac{Z_i\kappa_{0,e}^2\exp(-\kappa_D r)}{4\pi r}, \quad (60)$$

$$n_{D,i}^{(e)}(r) = N_i + \frac{\kappa_{0,i}^2\exp(-\kappa_D r)}{4\pi Z_i r}$$

where the screening constant κ_D is given by (57). From Eqs. (7), (8) and (58) it follows that DH potential

$\varphi_D^{(i,e)} = -Z_{i,e}e\kappa_D$ and consequently, DH potential energy $U_D^{(i,e)}$ is given by

$$U_D^{(i,e)} = -Z_{i,e}e\kappa_D = -\frac{(Z_{i,e}e)^2}{r_D}, \quad (61)$$

where κ_D and r_D are defined by Eq. (57).

APPENDIX B

The System $S_a^{(i)}$

The region $r_{s,i} < r < \infty$. We will start from the fact that the different electron-ion and dusty plasmas can be successfully described in the approximation of fixed heavy charged particles (see e.g. [11–15]). In accordance with this, we will treat the electronic component of the system $S_a^{(i)}$ considering all ions as immobile with respect to electrons, keeping in mind that in such a case the difference between the probe particle and ions does not exist regarding the electrons. In Fig. 2 the several of ions self-spheres (sphere with the radius $r_{s,i}$, centered on ions) in the neighborhood of the probe particle (the point O) are schematically shown. This figure should make more apparent the fact that the behavior of the electron inside the self-sphere of an ion in the point O' , far from the point O , has to be almost completely caused by its interaction with few ions which are close to the point O' . Because of that, in further considerations all ions are treated equally with the probe particle.

Here, we will denote by $(i; *)$ the possible ion configuration in the system $S_a^{(i)}$, and by $n_e^{(i;*)}(\mathbf{r})$ and $n_i^{(i;*)}(\mathbf{r})$ —the corresponding electron and ion densities. Also, we will denote by $\bar{n}_e^{(i;*)}(\mathbf{r})$ an average value of $\bar{n}_{e;out}^{(i;*)}$ within the part of space consisting of the outing of probe particle's and all ion's self-spheres (see shadowed area in Fig. 2). The condition of the thermodynamical equilibrium of the electron component in the case of a configuration $(i; *)$ has the form

$$\begin{aligned} \mu_e(n_e^{(i;*)}(\mathbf{r}), T_e) + (-e)\Phi^{(i;*)}(\mathbf{r}) \\ = \mu_e(n_e^{(i;*)}(\mathbf{r}_{st}^{i;*}), T_e) + (-e)\Phi^{(i;*)}(\mathbf{r}_{st}^{i;*}), \end{aligned} \quad (62)$$

where $\Phi^{(i;*)}(\mathbf{r})$ is the corresponding total electrostatic potential, and $\mathbf{r}_{st}^{(i;*)}$ is any point where $n_e^{(i;*)}(\mathbf{r}) = \bar{n}_{e;out}^{(i;*)}$. Finally, taking the chemical potentials $\mu_e(n_e^{(i;*)}(\mathbf{r}), T_e)$ in (62) as series of the differences $n_e^{(i;*)}(\mathbf{r}) - N_e$ along with keeping only first two members, we obtain $n_e^{(i;*)}(\mathbf{r})$ in the form

$$n_e^{(i;*)}(\mathbf{r}) = \bar{n}_{e; \text{out}}^{(i;*)} + \frac{-e}{(\partial\mu_e/\partial N_e)} \quad (63)$$

$$\times [\Phi^{(i;*)}(r_{\text{st}}^{(i;*)}) - \Phi^{(i;*)}(\mathbf{r})].$$

In contrast to (11), the equation (63) can be applied practically in the whole space, which is allowed by the behavior of the electron component in the presences of the positive charged particles.

In accordance with the described picture, the sought electron and ion densities $n_e^{(i)}(r)$ and $n_i^{(i)}(r)$ are treated here as a result of averaging of all densities $n_e^{(i;*)}(\mathbf{r})$ and $n_i^{(i;*)}(\mathbf{r})$. Because of the structure of Eq. (63) we will have that $n_e^{(i)}(r) = N_{e; \text{st}} + n_{e; d}(r)$, where $N_{e; \text{st}}$ and $n_{e; d}(r)$ are the mean values of the first and second members in Eq. (63). Taking the member $n_{e; d}(r)$ as series of the differences $[n_i^{(i)}(r) - N_i]$ along with keeping only first two members, we obtain that $n_e^{(i)}(r) = N_{e; \text{st}} + n_{e; d}(r = \infty) + K[n_i^{(i)}(r) - N_i]$, where accordingly to the boundary condition (3) the member $n_{e; d}(r = \infty)$ has to satisfy the equality $n_{e; d}(r = \infty) = N_e - N_{e; \text{st}}$. Finally, taking that $K = \alpha Z_i$ we obtain the mean electron density $n_e^{(i)}(r)$ in the form

$$n_e^{(i)}(r) = N_e + \alpha Z_i [n_i^{(i)}(r) - N_i]. \quad (64)$$

This relation shows that $n_e^{(i)}(r)$ in the region $r > r_{s; i} n_e^{(i)}(r)$ can be expressed by means only one unknown parameter α .

The region $0 < r < r_{s; i}$: the basic equation. Let $(s; *)$ denotes such an ion configuration in which the probe particle self-sphere is left free of ions, and $n_e^{(s;*)}(\mathbf{r})$ —the corresponding electron density. The member $n_{e; s}(r)$ in (22) will be identified with the result of averaging of densities $n_e^{(s;*)}(\mathbf{r})$ over all configurations $(s; *)$ in the region $0 < r \leq r_{s; i}$.

The corresponding condition of the thermodynamical equilibrium in the case of a configuration $(s; *)$ obtains from (62) replacing the index $(i, *)$ by $(s, *)$ and fixing the point $\mathbf{r}_{\text{st}}^{(s;*)} = \mathbf{r}_{\text{st}}$, where $r_{\text{st}} \leq r_{s; i}$. Then, repeating the procedure of the obtaining of Eq. (63), we obtain the equation

$$n_{e; s}(r) - n_{e; s}(r_{\text{st}}) = \frac{-e}{\partial\mu_e/\partial N_e} [\Phi_s(r_{\text{st}}) - \Phi_s(r)], \quad (65)$$

where the potentials $\Phi_s(r)$ and $\Phi_s(r_{\text{st}})$ in the considered region are given by Eq. (53) from Part 1 with the charge density $(-e)n_{e; s}(r)$. Since all information about the outgoing of the probe particle self-sphere ($r_{s; i} < r < \infty$) contained in the same constant members in expressions for

$\Phi_s(r)$ and $\Phi_s(r_{\text{st}})$, the difference $\Phi_s(r_{\text{st}}) - \Phi_s(r)$ in Eq. (65) is given by

$$\Phi_s(r_{\text{st}}) - \Phi_s(r) = \frac{Z_i e}{r_{\text{st}}} - \frac{Z_i e}{r}$$

$$- 4\pi \int_0^{r_{\text{st}}} (-e)n_{e; s}(r') \left(\frac{1}{r'} - \frac{1}{r_{\text{st}}} \right) r'^2 dr' \quad (66)$$

$$+ 4\pi \int_0^r (-e)n_{e; s}(r') \left(\frac{1}{r'} - \frac{1}{r} \right) r'^2 dr'.$$

From Eqs. (65) and (66) it follows the equation (26) for direct determination of the member $n_{e; s}(r)$. In order to find the solution of the equation (26), we will introduce the function $S(r)$ given by relation

$$n_s(r) - \frac{\kappa_{0; e}^2 Z_i}{4\pi r} = \frac{S(r)}{r}. \quad (67)$$

Consequently, Eq. (26), after the multiplication by r , transforms to the equation

$$S(r) - \kappa_{0; e}^2 \int_0^r S(r')(r-r') dr' - \frac{Z_i \kappa_{0; e}^4}{4\pi} \int_0^r (r-r') dr'$$

$$= \frac{r}{r_{\text{st}}} \left[S(r_{\text{st}}) - \kappa_{0; e}^2 \int_0^{r_{\text{st}}} S(r')(r_{\text{st}}-r') dr' \right. \quad (68)$$

$$\left. - \frac{Z_i \kappa_{0; e}^4}{4\pi} \int_0^{r_{\text{st}}} (r_{\text{st}}-r') dr' \right].$$

Applying the operator $\frac{d^2}{dr^2}$ to left and right sides of (68) we obtain the equation

$$\frac{d^2 S(r)}{dr^2} - \kappa_{0; e}^2 S(r) - \kappa_{0; e}^2 \frac{Z_i \kappa_{0; e}^2}{4\pi} = 0, \quad (69)$$

which can be presented in the form

$$\frac{d^2}{dr^2} \left[S(r) + \frac{Z_i \kappa_{0; e}^2}{4\pi} \right] = \kappa_{0; e}^2 \left[S(r) + \frac{Z_i \kappa_{0; e}^2}{4\pi} \right]. \quad (70)$$

From Eqs. (67) and (70) it follows that in the general case $n_s(r)$ is given by the relation

$$n_s(r) = \frac{A \exp(-\kappa_{0; e} r) + B \exp(\kappa_{0; e} r)}{r}. \quad (71)$$

Taking the coefficients A and B in the form

$$A = r_{s; i} N_e a, \quad B = r_{s; i} N_e b, \quad (72)$$

we will present Eq. (26) as

$$a \exp(-\kappa_{0; e} r) + b \exp(\kappa_{0; e} r) - \frac{Z_i \kappa_{0; e}^2}{4\pi r_{s; i} N_e}$$

$$\begin{aligned}
& + \kappa_{0;e}^2 \int_0^r [a \exp(-\kappa_{0;e} r') + b \exp(\kappa_{0;e} r')] (r-r') dr' \\
& = \frac{r}{r_{st}} \left\{ a \exp(-\kappa_{0;e} r_{st}) + b \exp(\kappa_{0;e} r_{st}) - \frac{Z_i \kappa_{0;e}^2}{4\pi r_{s;i} N_e} \right. \\
& \left. + \kappa_{0;e}^2 \int_0^{r_{st}} [a \exp(-\kappa_{0;e} r') + b \exp(\kappa_{0;e} r')] (r_{st}-r') dr' \right\}.
\end{aligned} \quad (73)$$

From here it follows the equation

$$\begin{aligned}
& (b-a)\kappa_{0;e} r + \left(a + b - \frac{Z_i \kappa_{0;e}^2}{4\pi r_{s;i} N_e} \right) \\
& = \frac{r}{r_{st}} \left[(b-a)\kappa_{0;e} r_{st} + \left(a + b - \frac{Z_i \kappa_{0;e}^2}{4\pi r_{s;i} N_e} \right) \right],
\end{aligned} \quad (74)$$

which can be presented in the form

$$a + b - \frac{Z_i \kappa_{0;e}^2}{4\pi r_{s;i} N_e} = \frac{r}{r_{st}} \left(a + b - \frac{Z_i \kappa_{0;e}^2}{4\pi r_{st} N_e} \right), \quad (75)$$

where in the general case $r \neq r_{st}$. This means that the coefficients a and b have to satisfy the condition

$$a + b = \frac{Z_i \kappa_{0;e}^2}{4\pi r_{s;i} N_e} \equiv \frac{(\kappa_{0;e} r_{s;i})^2}{3}, \quad (76)$$

which provides that unknown parameter r_{st} disappears from further considerations, and that $\rho_s(r)$ given by Eqs. (71) and (72) really satisfies the equation (26). Then, from Eqs. (71) and (72) it follows the sought expression (28) for the member $n_{e;s}(r)$. Finally, from Eqs. (25) and (28) it is obtained the other condition which is necessary for the determination a and b , namely

$$a \exp(-\kappa_{0;e} r_{s;i}) + b \exp(\kappa_{0;e} r_{s;i}) = 1 - \alpha, \quad (77)$$

where the free parameter α has to be determined from the condition (23).

APPENDIX C

The System $S_a^{(e)}$

The region: $r_{s;e} < r < \infty$. In the case (e) we will start from the fact that the fixed probe particle with the charge ($-e$) *principally can not be represented as a free electron in the system S_{in} from the aspect of the interaction with positive ions*, since their average distribution in the neighborhood of such a particle would resemble to a distribution of positive ions in plasma in the neighborhood of a heavy negative ion.

The exit from this situation is the treatment of accessory system $S_a^{(e)}$ as a single component one with the electron gas on the corresponding positive charged background, which, contrary to the background described in Part 1, is not homogeneous. We keep in mind the back-

ground with the charge density taken in the form $Z_i e n_i^{(e)}(r)$, which in the classical case is able to model the average distribution of positive charge in the neighborhood of an electron in the system S_{in} . If sought distribution is found, the factor $n_i^{(e)}(r)$ could be treated as the corresponding ion density. Here we use the fact that such a distribution can be found since existing conditions which establish correspondence between the systems $S_a^{(e)}$ and S_{in} are enough for determination of all needed parameters.

One of the mentioned conditions is that the behavior of $n_e^{(e)}(r)$ and $n_i^{(e)}(r)$ reflect the existence of the electron-ion correlation (discussed in Section 3.1) which is characterized by the coefficient α . We will take into account that this coefficient in the case (e) can be treated as the probability of the following event: the decreasing of number of electrons for Z_i in the region $r_{s;e} < r < \infty$ coincides with the decreasing of the number of ions for unity. This means that in this case we have the relation

$$\alpha = \frac{[N_e - n_e^{(e)}(r)]/Z_i}{N_i - n_i^{(e)}(r)}, \quad (78)$$

which connects $n_e^{(e)}(r)$ and $n_i^{(e)}(r)$ in the region $r_{s;e} < r < \infty$ by means of the known parameter α . In Section 3.2 the relation (78) is taken as the starting point for the determination $n_i^{(e)}(r)$ and $n_e^{(e)}(r)$ outside of probe particle self-sphere in the system $S_a^{(e)}$.

The region: $0 < r < r_{s;e}$. In order to determine the member $n_{i;s}(r)$ in Eq. (40) we will take into account the following facts: (1) the form of $n_{i;s}(r)$ has to be similar to the form of the member $n_{e;s}(r)$ in the case (i); (2) the parameters which characterize $n_{i;s}(r)$ have to be closely connected with the parameters which characterize $n_{e;s}(r)$; (3) the procedure of obtaining of $n_{i;s}(r)$ has to provide automatic applicability for all possible N_e , Z_i , T_e and T_i and (4) self-consistence of final expressions. In accordance with this, the member $n_{i;s}(r)$ will be taken as a superposition of two dimensionless functions of dimensionless argument ($r/r_{s;e}$), given by relations

$$\begin{aligned}
n_{i;s}(r) & = a_i (r/r_{s;e})^{-1} \exp[-(\kappa_{s;i} r_{s;e}) r/r_{s;e}] \\
& + b_i (r/r_{s;e})^{-1} \exp[(\kappa_{s;i} r_{s;e}) r/r_{s;e}],
\end{aligned} \quad (79)$$

$$\kappa_{s;i} = \kappa_{0;e} \frac{r_{s;i}}{r_{s;e}} = \kappa_{0;e} Z_i^{\frac{1}{3}}, \quad (80)$$

where $\kappa_{0;e}$ is given by (27), and the screening constant $\kappa_{s;i}$ is chosen in such a manner which guarantees that the relation

$$\frac{r'}{r_{s;i}} \exp\left(\mp \kappa_{s;i} r_{s;e} \frac{r'}{r_{s;i}}\right) = \frac{r''}{r_{s;e}} \exp\left(\mp \kappa_{0;e} r_{s;i} \frac{r''}{r_{s;e}}\right) \quad (81)$$

is valid for any $(r'/r_{s_i}; i) = (r''/r_{s_i}; e)$, when $0 < r' < r_{s_i}; i$ and $0 < r'' < r_{s_i}; e$. The coefficients a_i and b_i in Eq. (79) have to be found from the conditions (41) and (42).

APPENDIX D

The Interpretation of the Systems Considered

Let $S_{\text{in}; M_i}$ be the model finite spherical system with total ion and electron numbers M_i and $M_e = Z_i M_i$, where M_i is an integer number, and with the radius R_{M_i} and volume V_{M_i} determined by relation: $M_i/V_{M_i} = N_i$. From here it follows that the system $S_{\text{in}; M_i}$ is neutral as a whole and $M_e/V_{M_i} = N_e = Z_i N_i$. We will assume that in $S_{\text{in}; M_i}$ the ion and electron components can be treated as gases in states of the thermodynamical equilibrium with temperatures T_i and T_e . In a usual way, we will treat the basic system S_{in} as a thermodynamical limit of the systems $S_{\text{in}; M_i}$, i.e. as the result of transition: $M_i \rightarrow \infty$ and $V_{M_i} \rightarrow \infty$, under conditions

$$M_i/V_{M_i} = N_i, \quad T_i = \text{const}, \quad T_e = \text{const}. \quad (82)$$

In the case (i), we will associate with every system $S_{\text{in}; M_i}$ an other system $S_{a; M_i}^{(i)}$, which differs from $S_{\text{in}; M_i}$ only by the change of one of the free ions for the probe particle, with the same charge $Z_i e$ fixed in the center of that system.

We will take into account that the systems $S_{a; M_i}^{(i)}$ are also neutral as a whole. From here follows the relation

$$Z_i e + Z_i e \int_0^{R_{M_i}} n_{i; M_i}(r) \times 4\pi r^2 dr - e N_e V_{M_i} = 0, \quad (83)$$

where $n_{i; M_i}(r)$ are the corresponding mean local ion densities, and

$$N_e V_{M_i} = Z_i N_i \frac{4\pi}{3} R_{M_i}^3 \equiv Z_i \int_0^{R_{M_i}} N_i \times 4\pi r^2 dr. \quad (84)$$

From (83) and (84), after their multiplication with $(Z_i e)^{-1}$ the equation follows

$$\int_0^{R_{M_i}} [N_i - n_{i; M_i}(r)] \times 4\pi r^2 dr = 1. \quad (85)$$

Based on it, we have it that

$$\begin{aligned} \lim_{M_i \rightarrow \infty} \int_0^{R_{M_i}} [N_i - n_{i; M_i}(r)] \times 4\pi r^2 dr \\ = \int_0^{\infty} [N_i - n_{i; \infty}(r)] \times 4\pi r^2 dr = 1, \end{aligned} \quad (86)$$

where $n_{i; \infty}(r) = \lim_{M_i \rightarrow \infty} n_{i; M_i}(r)$ for any $r \geq 0$. Here, we will treat just $n_{e; \infty}(r)$ as the mean local ion density in the accessory systems $S_a^{(i)}$, i.e. as $n_i^{(i)}(r)$. In accordance with this, from (86) it directly follows that $n_i^{(i)}(r)$ has to satisfy the condition (13) for the case (i).

Since a similar reasoning may be repeated in the case (e), we can consider the condition (13) must be satisfied in both (i) and (e) cases. Finally, it is clear that in the case when one of components in described systems is changed with the corresponding background, we obtain the equation which corresponds to Eq. (21) from Part 1.

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