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Spectral coefficients of emission and absorption due to ion-atom radiation collisions in the solar atmosphere*

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Abstract. — Spectral coefficients of spontaneous emission and absorption (for 365 nm $\leq \lambda \leq$820 nm range) due to ion-atom radiation processes $H^+ + H(1s) \leftrightarrow H_2^+ (1\Sigma_g)$ and $H^+ + H(1s) \leftrightarrow H(1s) + H^+$ are presented. Calculations have been performed within semiclassical approach for standard solar photosphere and chromosphere models. The presented numerical results enable the inclusion of considered ion-atom radiative processes in the optical depth calculation for the layers mentioned. These results might be of interest as well for other astrophysical plasmas with dominant hydrogen component and temperatures around 6000 K.

Key words: sun: atmosphere — radiation mechanisms: thermal — atomic data — atomic processes

1. Introduction

We present here results of our calculations of the contribution of ion-atom radiative processes:

$$H^+ + H(1s) \leftrightarrow \varepsilon_\lambda + H_2^+ (1\Sigma_g)\,, \qquad (1a)$$

$$H^+ + H(1s) \leftrightarrow \varepsilon_\lambda + H(1s) + H^+, \qquad (1b)$$

to the solar photosphere and chromosphere continuous spectra in the optical range. Here ε_λ is the energy of photon with wavelength λ, where 365 nm $\leq \lambda \leq$ 820 nm. Our calculations have been made within semiclassical approach (Drukarev & Mihajlov 1974; Mihajlov & Popović 1981; Mihajlov & Dimitrijević 1986). It was demonstrated recently (Mihajlov et al. 1993) that the influence of processes (1a, b), neglected up to now, is significant in some photospheric and chromospheric layers.

In this paper, the calculations of the contribution of processes (1a,b) to the solar continuous spectra, for the commonly used photospheric (Maltby et al. 1986, their Table 11) and chromospheric (Vernazza et al. 1981, their model C) models are presented. We provide data for the whole height range for both models in order to avoid inconveniences due to artificial interruption near the fitting point. In the considered range we compare the contribution of processes (1a,b) with the contribution of electron-ion and electron-atom radiative processes, including the processes of creation and photodissociation of H^- ion (Mihalas 1978). The particular contribution of process (1a) and (1b) has been determined separately.

2. Basic relations

Spectral characteristics of the solar plasma, related to ion-atom processes (1a) and (1b), i.e. partial spectral emissivity $\varepsilon_{ia}^{(a,b)}(\lambda)$ and partial spectral absorption coefficient $\kappa_{ia}^{(a,b)}(\lambda)$ will be searched in the form

$$\varepsilon_{ia}^{(a,b)}(\lambda) = S_{ia}^{(a,b)}(\lambda) N\left(H^+\right) N(H),$$
$$\kappa_{ia}^{(a,b)}(\lambda) = K_{ia}^{(a,b)}(\lambda) N\left(H^+\right) N(H) \qquad (2)$$

where $N\left(H^+\right)$ and $N(H)$ are proton (H^+) and atom H(1s) densities. The total spectral emissivity and total spectral absorption coefficient, i.e.

$$\varepsilon_{ia}(\lambda) = \varepsilon_{ia}^{(a)}(\lambda)+\varepsilon_{ia}^{(b)}(\lambda) \quad \text{and} \quad \kappa_{ia}(\lambda) = \kappa_{ia}^{(a)}(\lambda)+\kappa_{ia}^{(b)}(\lambda),$$

will be presented here in the same form:

$$\varepsilon_{ia}(\lambda) = S_{ia}(\lambda) N\left(H^+\right) N(H),$$
$$\kappa_{ia}(\lambda) = K_{ia}(\lambda) N\left(H^+\right) N(H), \qquad (3)$$

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* Tables 1.1; 1.2; 2.1; 2.2; 3.1 and 3.2 are also available in electronic form: see the editorial in A&A 1992, Vol. 266, No. 2, page E1

where

$$S_{\rm ia}(\lambda) = S_{\rm ia}^{(\rm a)}(\lambda) + S_{\rm ia}^{(\rm b)}(\lambda),$$
$$K_{\rm ia}(\lambda) = K_{\rm ia}^{(\rm a)}(\lambda) + K_{\rm ia}^{(\rm b)}(\lambda); \qquad (4)$$

The spontaneous emission spectral coefficient $S_{\rm ia}$ characterizes the total contribution of the photoassociation process (1a) and photoemission charge exchange process (1b). Similarly, the absorption spectral coefficient $K_{\rm ia}$ characterizes the total contribution of the photodissociation process (1a) and the photoabsorption charge exchange process (1b).

The coefficients $S_{\rm ia}$ and $K_{\rm ia}$ have been determined within the semiclassical approximation by Mihajlov & Popović (1981) and Mihajlov & Dimitrijević (1986) and may be expressed as

$$S_{\rm ia}(\lambda, T) = 4.7769\ 10^{-34}\frac{C\left(R_\lambda\right)\left(R_\lambda/a_0\right)^4}{1-a_0/R_\lambda}\left(\frac{\varepsilon_\lambda}{2R_y}\right)^5$$
$$\exp\left[-\frac{U_2\left(R_\lambda\right)}{kT}\right], \qquad (5)$$
$$K_{\rm ia}(\lambda, T) = 0.620\ 10^{-42}\frac{C\left(R_\lambda\right)\left(R_\lambda/a_0\right)^4}{1-a_0/R_\lambda}\theta(\lambda, T)$$
$$\exp\left[-\frac{U_1\left(R_\lambda\right)}{kT}\right], \qquad (6)$$

where T is the plasma temperature and

$$C\left(R_\lambda\right) = \left[\frac{2D_{12}\left(R_\lambda\right)}{{\rm e}\,R_\lambda}\right]^2 \times \frac{1-a_0/R_\lambda}{\gamma\left(R_\lambda\right)},$$
$$\theta(\lambda, T) = 1 - \exp\left(-\frac{\varepsilon_\lambda}{kT}\right),$$
$$\gamma\left(R_\lambda\right) = \left|\frac{{\rm d}\ln\left[E_{12}(R)/2R_y\right]}{{\rm d}\left(R/a_0\right)}\right|_{R=R_\lambda},$$
$$E_{12}(R) = U_2(R) - U_1(R). \qquad (7)$$

Here, R is the distance between protons (in the system $H^+ + H$), $U_1(R)$ and $U_2(R)$ - adiabatic terms of the ground ($1\Sigma_g$) and the first excited ($1\Sigma_u$) electronic states of the ion H_2^+, and $D_{12}(R)$ - module of dipole matrix element (between these states). Finally, R_λ is the root of the equation:

$$E_{12}(R) = \varepsilon_\lambda, \qquad (8)$$

where $\varepsilon_\lambda = 2\pi\hbar c/\lambda$. The adiabatic terms $U_{1,2}(R)$ and the matrix element $D_{12}(R)$ are tabulated (as functions of R) in Bates et al. (1953) and Ramaker & Peak (1973), respectively.

With given numerical coefficients in Eqs.(5) and (6) spectral coefficients $S_{\rm ia}(\lambda, T)$ and $K_{\rm ia}(\lambda, T)$ are expressed here in $\left[J\ {\rm cm}^3\ {\rm s}^{-1}\ {\rm nm}^{-1}\right]$ and $\left[{\rm cm}^{-1}\right]$ units, respectively.

We will take here the partial spectral coefficients $S_{\rm ia}^{(\rm a,b)}$ and $K_{\rm ia}^{(\rm a,b)}$ in the form:

$$S_{\rm ia}^{(\rm a,b)} = S_{\rm ia}X^{(\rm a,b)}, \quad K_{\rm ia}^{(\rm a,b)} = K_{\rm ia}^{(\rm a,b)}X^{(\rm a,b)}, \qquad (9)$$

where

$$X^{(\rm a)} + X^{(\rm b)} \equiv 1. \qquad (10)$$

According to Mihajlov & Dimitrijević (1986), parameters $X^{(\rm a,b)}$ may be expressed as

$$X^{(\rm a)}(Z) = \frac{\gamma(3/2; Z)}{\Gamma(3/2)}, \quad X^{(\rm b)}(Z) = \frac{\Gamma(3/2; Z)}{\Gamma(3/2)}, \quad (\text{for } Z \geq 0)$$
$$X^{(\rm a)}(Z) = 0, \quad X^{(\rm b)}(Z) = 1, \quad (\text{for } Z \leq 0) \qquad (11)$$

where

$$Z = -\frac{U_1\left(R_\lambda\right)}{kT}, \qquad (12)$$

and $\gamma(3/2; Z)$, $\Gamma(3/2; Z)$ are incomplete Gamma functions. In the considered λ range we have $U_1\left(R_\lambda\right) < 0$ and $Z = \left|U_1\left(R_\lambda\right)\right|/kT$.

Since according to Eq. (10) it is sufficient to determine only one of these parameters, we will use the parameter $X^{(\rm b)}$ characterizing only radiative charge exchange, i.e. process (1b), in accordance with our previous article (Mihajlov & Dimitrijević 1986).

Parameters R_λ and $U_{1,2}\left(R_\lambda\right)$, needed for the calculation of $S_{\rm ia}$ and $K_{\rm ia}$ for 250 nm $\leq \lambda \leq$ 950 nm range, are tabulated in Mihajlov et al. (1983). In this λ range R_λ and $U_2\left(R_\lambda\right)$ may be fitted well with the following expressions:

$$R_\lambda = 1.37347\ln\lambda - 4.41481, \quad U_2\left(R_\lambda\right) = 90.5631\,\lambda^{-1.20962}, \qquad (13)$$

where R_λ and $U_2\left(R_\lambda\right)$ are expressed in atomic units and λ in nm.

The function $X^{(\rm b)}(Z)$ is tabulated in Mihajlov & Dimitrijević 1986, for $0.6 \leq \sqrt{Z} \leq 3.0$ range. As shown in this Ref., $X^{(\rm b)}(Z)$ may be expressed in the form

$$X^{(\rm b)}(Z) = 1 - \Phi\left(\sqrt{Z}\right) + \left(2/\sqrt{\pi}\right)\cdot\sqrt{Z}\exp(-Z), \qquad (14)$$

where Φ is the error function (Abramowitz & Stegun 1972).

3. Results and discussion

Our calculations have been based on standard photospheric model of Maltby et al. (1986) and standard chromospheric model of Vernazza et al. (1981, model C). Plasma parameters T, $N_{\rm e}$, N(H) and $N\left(H^+\right)$ for the mentioned models are presented in tables 1.1 and 1.2 as functions of height (h). In these tables are presented as well parameters

$$\eta_{\rm ea} = \frac{N\left(H^+\right)}{N_{\rm e}} \quad \text{and} \quad \eta_{\rm ei} = \eta_{\rm ea}\cdot\frac{N(H)}{N_{\rm i}}, \qquad (15)$$

where N_i is the total density of positive atomic ions. They help to represent relative contributions of (1a,b) processes compared to electron-ion (η_{ei}) and electron-atom (η_{ea}) processes separately (see Mihajlov et al. 1993).

Spectral coefficients $S_{ia}(\lambda, T)$ and $K_{ia}(\lambda, T)$ as function of T for different λ are shown in Figs. 1 and 2 for $T \leq 10^4$K. The behavior of factor $X^b(Z)$ is illustrated in Fig. 3.

Emissivity $\varepsilon_{ia}(\lambda, T)$ and absorption coefficient $\kappa_{ia}(\lambda, T)$ are presented in Tables 2.1, 2.2 and 3.1, 3.2, respectively. Tables 2.1, 3.1 correspond to photospheric, and 2.2, 3.2 to chromospheric models. In these tables the temperature range is limited to $T < 10^4$K and the wavelength range is 365 nm $\leq \lambda \leq$ 820 nm. The presented tables enable direct inclusion of the influence of (1a,b) processes for calculations of spectral characteristics (first of all the optical depth) of the considered photospheric and chromospheric layers, within standard solar atmosphere models used here.

The relative contribution of (1a) and (1b) processes together, compared to the total electron-atom and electron-ion processes contribution is illustrated in Figs. 4.1 and 4.2. These figures show the change of

$$F = \frac{\varepsilon_{ia}}{\varepsilon_{ei} + \varepsilon_{ea}} \qquad (16)$$

quantity as a function of height (h), where ε_{ei} and ε_{ea} are total contributions of free-free and free-bound transitions during electron-ion and electron-atom scattering respectively. If LTE exists, the quantity F represents the ratio of corresponding absorption coefficients as well. We assume here that

$$\varepsilon_{ei} = \xi_{ei} S_{ei} N_e^2, \quad \varepsilon_{ea} = \xi_{ea} S_{ea} N(\mathrm{H}) N_e, \qquad (17)$$

where the corresponding spectral coefficients S_{ei} and S_{ea} have been determined for the purely hydrogenic plasma and the Biberman-Norman factor ξ_{ei} as well as the factor ξ_{ea} take into account the real composition of photospheric and chromospheric plasma, i.e. the presence of Na^+, Mg^+, Al^+, Ca^+ ions as well as the presence of helium atoms.

Figures 4.1 and 4.2 show that F, as a function of h, has a minimum in 100 km $< h <$ 600 km range which coincides with the temperature minimum (see Tables 1.1 and 1.2). The minimum of F appears because the relative hydrogen contribution to the free electron density in this range is very small (Vernazza et al. 1981).

The behavior of $X^{(b)}$ parameter, defined by Eqs. (9-14), as a function of h, is presented in Figs. 5.1 and 5.2. These figures show that $X^{(b)}$ has a minimum which coincides with the minimum of function F. This minimum is the result of $X^{(b)}(Z)$ dependency (see Eq. (11) and Fig. 3) where is $Z \sim 1/T$ (see Eq. (12)).

In Figs. 4.1 and 4.2 the ranges where processes (1a,b) are of influence (e.g. where the value of the quantity F is 0.04 – 0.12) are shown. Moreover, in Figs. 5.1 and 5.2, is shown that outside the photospheric minimum surroundings, the influence of the radiation charge exchange processes (1b), due to H^+ and H(1s) collisions, is comparable and somewhere surpasses the influence of (1a), i.e. photoassociation H^+ and H(1s) and photodissociation of molecular ion H_2^+ $(1\Sigma_g)$.

Figures 4.1, 5.1 concern the photospheric and 4.2, 5.2 the chromospheric model. We can see that within both models, the results obtained are qualitatively similar but for particular layers quantitative differences exist as expected.

4. Conclusion

These results illustrate the influence of (1a,b) radiative processes involving H^+ and H(1s) on spectral characteristics for particular photospheric and chromospheric layers. The numerical results presented enable the inclusion of ion-atom radiative processes (1a,b) in the optical depth calculation for the mentioned layers. Our results might be of interest as well for other astrophysical plasmas with a dominant hydrogen component and temperatures around 6000 K.

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Table 1.1. Basic plasma parameters (temperature T, electron density N_e, total hydrogen density $N(\mathrm{H})$) as well as parameter η_{ei} and η_{ea} defined by Eq. (15), for solar photospheric model of Maltby et al. (1986), as a function of height h

h[km]	T[K]	N_e[cm^{-3}]	N(H)[cm^{-3}]	η_{ei}	η_{ea}
2040	7360	.4745E +11	.1068E +12	.122E +01	.100E +01
1948	6940	.5303E +11	.1611E +12	.203E +01	.100E +01
1810	6630	.4475E +11	.3165E +12	.606E +01	.100E +01
1634	6440	.4768E +11	.7729E +12	.152E +02	.100E +01
1406	6280	.7199E +11	.2706E +13	.366E +02	.100E +01
1198	6150	.1118E +12	.9317E +13	.823E +02	.100E +01
1079	6040	.1414E +12	.2043E +14	.143E +03	.100E +01
912	5755	.1624E +12	.6686E +14	.411E +03	.100E +01
862	5650	.1649E +12	.9819E +14	.594E +03	.100E +01
761	5280	.1298E +12	.2246E +15	.173E +04	.100E +01
711	5030	.1078E +12	.3553E +15	.206E +04	.624E +00
655	4750	.1030E +12	.6014E +15	.975E +03	.167E +00
605	4550	.1321E +12	.9871E +15	.271E +03	.363E -01
553	4410	.1976E +12	.1642E +16	.711E +02	.856E -02
528	4400	.2445E +12	.2090E +16	.559E +02	.654E -02
503	4400	.3040E +12	.2654E +16	.469E +02	.537E -02
478	4410	.3793E +12	.3365E +16	.422E +02	.476E -02
453	4460	.4714E +12	.4218E +16	.525E +02	.587E -02
428	4510	.5886E +12	.5273E +16	.635E +02	.708E -02
403	4560	.7337E +12	.6579E +16	.761E +02	.849E -02
378	4610	.9128E +12	.8168E +16	.901E +02	.101E -01
352	4660	.1133E +13	.1016E +17	.107E +03	.119E -01
301	4770	.1733E +13	.1556E +17	.159E +03	.177E -01
250	4880	.2642E +13	.2350E +17	.223E +03	.251E -01
200	4990	.3997E +13	.3502E +17	.302E +03	.344E -01
175	5060	.4939E +13	.4236E +17	.370E +03	.431E -01
150	5150	.6142E +13	.5088E +17	.491E +03	.593E -01
125	5270	.7751E +13	.6052E +17	.720E +03	.922E -01
100	5410	.1001E +14	.7136E +17	.105E +04	.147E +00
75	5580	.1352E +14	.8328E +17	.148E +04	.240E +00
50	5790	.1977E +14	.9598E +17	.185E +04	.381E +00
35	5980	.2772E +14	.1031E +18	.193E +04	.520E +00
20	6180	.4052E +14	.1102E +18	.175E +04	.642E +00
10	6340	.5484E +14	.1146E +18	.151E +04	.721E +00
0	6520	.7676E +14	.1187E +18	.122E +04	.791E +00
-10	6720	.1104E +15	.1224E +18	.940E +03	.848E +00
-20	6980	.1725E +15	.1250E +18	.652E +03	.901E +00
-30	7280	.2798E +15	.1269E +18	.426E +03	.941E +00
-40	7590	.4463E +15	.1285E +18	.277E +03	.966E +00
-50	7900	.6897E +15	.1300E +18	.184E +03	.982E +00
-60	8220	.1046E +16	.1312E +18	.124E +03	.994E +00
-70	8540	.1540E +16	.1322E +18	.848E +02	.100E +01
-80	8860	.2207E +16	.1330E +18	.592E +02	.100E +01
-90	9140	.2970E +16	.1343E +18	.442E +02	.100E +01
-100	9400	.3856E +16	.1357E +18	.342E +02	.100E +01

Table 1.2. Same as in Table 1.1. but for the chromospheric model of Vernazza et al. (1981, model C)

h[km]	T[K]	N_e[cm^{-3}]	N(H)[cm^{-3}]	η_{ei}	η_{ea}
2016	7360	.3811E +11	.9075E +11	.135E +01	.100E +01
1990	7160	.3858E +11	.1033E +12	.166E +01	.100E +01
1925	6940	.4028E +11	.1380E +12	.241E +01	.100E +01
1785	6630	.4771E +11	.2601E +12	.444E +01	.100E +01
1605	6440	.6005E +11	.6386E +12	.963E +01	.100E +01
1515	6370	.6456E +11	.1048E +13	.152E +02	.100E +01
1380	6280	.7600E +11	.2273E +13	.289E +02	.100E +01
1280	6220	.7486E +11	.4200E +13	.551E +02	.100E +01
1180	6150	.8108E +11	.7865E +13	.960E +02	.100E +01
1065	6040	.9349E +11	.1711E +14	.182E +03	.100E +01
980	5925	.1041E +12	.3147E +14	.301E +03	.100E +01
905	5755	.1049E +12	.5546E +14	.528E +03	.100E +01
855	5650	.1064E +12	.8135E +14	.764E +03	.100E +01
755	5280	.8838E +11	.1864E +15	.211E +04	.100E +01
705	5030	.7664E +11	.2935E +15	.383E +04	.100E +01
655	4730	.8085E +11	.4794E +15	.111E +04	.187E +00
605	4420	.1112E +12	.8119E +15	.106E +03	.145E -01
555	4230	.1733E +12	.1382E +16	.153E +02	.192E -02
515	4170	.2495E +12	.2096E +16	.675E +01	.803E -03
450	4220	.4516E +12	.3989E +16	.657E +01	.744E -03
350	4465	.1110E +13	.9979E +16	.235E +02	.261E -02
250	4780	.2674E +13	.2315E +17	.103E +03	.119E -01
150	5180	.6476E +13	.4917E +17	.472E +03	.621E -01
100	5455	.1066E +14	.6866E +17	.104E +04	.161E +00
50	5840	.2122E +14	.9203E +17	.176E +04	.406E +00
0	6420	.6433E +14	.1166E +18	.134E +04	.741E +00
-25	6910	.1547E +15	.1261E +18	.721E +03	.885E +00
-50	7610	.4645E +15	.1317E +18	.274E +03	.969E +00
-75	8320	.1204E +16	.1365E +18	.112E +03	.100E +01

Table 2.1. Emissivity $\varepsilon_{ia}(\lambda)$ due to (1a) and (1b) reactions together as a function of λ and height h, for the photospheric model of Maltby et al. (1986)

h[km]	$\varepsilon_{1a}(\lambda)$; λ [nm], ε_{1a} [$cm^{-3}s^{-1}$• J/nm]					
	365	400	500	600	700	820
2040	.426E -15	.427E -15	.368E -15	.285E -15	.213E -15	.149E -15
1948	.731E -15	.747E -15	.669E -15	.532E -15	.405E -15	.287E -15
1810	.134E -14	.139E -14	.128E -14	.104E -14	.804E -15	.577E -15
1634	.344E -14	.360E -14	.341E -14	.280E -14	.218E -14	.158E -14
1406	.172E -13	.182E -13	.176E -13	.146E -13	.115E -13	.839E -14
1198	.866E -13	.924E -13	.905E -13	.762E -13	.603E -13	.443E -13
1079	.226E -12	.243E -12	.241E -12	.205E -12	.163E -12	.120E -12
912	.707E -12	.775E -12	.801E -12	.698E -12	.566E -12	.424E -12
862	.981E -12	.108E -11	.114E -11	.100E -11	.816E -12	.615E -12
761	.133E -11	.152E -11	.169E -11	.154E -11	.129E -11	.997E -12
711	.882E -12	.103E -11	.120E -11	.113E -11	.963E -12	.754E -12
655	.293E -12	.350E -12	.432E -12	.421E -12	.369E -12	.295E -12
605	.108E -12	.133E -12	.171E -12	.172E -12	.153E -12	.125E -12
553	.543E -13	.675E -13	.901E -13	.924E -13	.838E -13	.690E -13
528	.646E -13	.804E -13	.108E -12	.110E -12	.100E -12	.827E -13
503	.837E -13	.104E -12	.139E -12	.143E -12	.130E -12	.107E -12
478	.119E -12	.148E -12	.197E -12	.202E -12	.183E -12	.151E -12
453	.242E -12	.299E -12	.394E -12	.401E -12	.362E -12	.296E -12
428	.482E -12	.592E -12	.772E -12	.779E -12	.699E -12	.571E -12
403	.950E -12	.116E -11	.149E -11	.150E -11	.134E -11	.109E -11
378	.184E -11	.223E -11	.284E -11	.283E -11	.251E -11	.203E -11
352	.354E -11	.428E -11	.539E -11	.532E -11	.470E -11	.379E -11
301	.138E -10	.164E -10	.202E -10	.196E -10	.172E -10	.137E -10
250	.500E -10	.590E -10	.709E -10	.680E -10	.588E -10	.466E -10
200	.172E -09	.200E -09	.235E -09	.223E -09	.191E -09	.150E -09
175	.342E -09	.397E -09	.460E -09	.432E -09	.368E -09	.288E -09
150	.761E -09	.875E -09	.998E -09	.926E -09	.784E -09	.609E -09
125	.196E -08	.223E -08	.249E -08	.228E -08	.191E -08	.147E -08
100	.533E -08	.600E -08	.654E -08	.590E -08	.489E -08	.374E -08
75	.156E -07	.173E -07	.183E -07	.163E -07	.133E -07	.101E -07
50	.484E -07	.529E -07	.544E -07	.472E -07	.382E -07	.285E -07
35	.113E -06	.121E -06	.122E -06	.104E -06	.830E -07	.614E -07
20	.246E -06	.262E -06	.255E -06	.214E -06	.169E -06	.124E -06
10	.426E -06	.450E -06	.430E -06	.357E -06	.279E -06	.203E -06
0	.748E -06	.781E -06	.732E -06	.599E -06	.465E -06	.335E -06
-10	.132E -05	.136E -05	.125E -05	.101E -05	.774E -06	.554E -06
-20	.254E -05	.258E -05	.231E -05	.183E -05	.139E -05	.985E -06
-30	.498E -05	.500E -05	.434E -05	.338E -05	.254E -05	.178E -05
-40	.938E -05	.929E -05	.785E -05	.601E -05	.445E -05	.309E -05
-50	.167E -04	.164E -04	.135E -04	.102E -04	.745E -05	.512E -05
-60	.289E -04	.280E -04	.225E -04	.167E -04	.121E -04	.826E -05
-70	.477E -04	.456E -04	.359E -04	.263E -04	.189E -04	.128E -04
-80	.753E -04	.714E -04	.550E -04	.397E -04	.283E -04	.190E -04
-90	.110E -03	.103E -03	.784E -04	.560E -04	.396E -04	.264E -04
-100	.154E -03	.143E -03	.107E -03	.758E -04	.533E -04	.353E -04

Table 2.2. Same as in Table 2.1. but for the chromospheric model of Vernazza et al. (1981, model C)

h[km]	$\varepsilon_{1a}(\lambda)$; λ [nm], ε_{1a} [$cm^{-3}s^{-1}$• J/nm]					
	365	400	500	600	700	820
2016	.303E -15	.303E -15	.261E -15	.202E -15	.151E -15	.106E -15
1990	.350E -15	.353E -15	.310E -15	.243E -15	.183E -15	.129E -15
1925	.502E -15	.512E -15	.459E -15	.365E -15	.278E -15	.197E -15
1785	.111E -14	.115E -14	.107E -14	.867E -15	.670E -15	.481E -15
1605	.345E -14	.362E -14	.342E -14	.282E -14	.220E -14	.159E -14
1515	.607E -14	.639E -14	.609E -14	.504E -14	.394E -14	.286E -14
1380	.152E -13	.161E -13	.155E -13	.129E -13	.101E -13	.739E -14
1280	.271E -13	.288E -13	.279E -13	.234E -13	.184E -13	.135E -13
1180	.531E -13	.567E -13	.555E -13	.467E -13	.370E -13	.271E -13
1065	.125E -12	.134E -12	.134E -12	.114E -12	.904E -13	.667E -13
980	.239E -12	.259E -12	.261E -12	.224E -12	.180E -12	.133E -12
905	.379E -12	.415E -12	.430E -12	.374E -12	.303E -12	.227E -12
855	.524E -12	.579E -12	.608E -12	.535E -12	.437E -12	.329E -12
755	.753E -12	.857E -12	.955E -12	.873E -12	.732E -12	.563E -12
705	.831E -12	.966E -12	.113E -11	.106E -11	.906E -12	.710E -12
655	.200E -12	.240E -12	.298E -12	.291E -12	.256E -12	.205E -12
605	.260E -13	.322E -13	.429E -13	.439E -13	.398E -13	.328E -13
555	.722E -14	.918E -14	.128E -13	.136E -13	.126E -13	.105E -13
515	.611E -14	.783E -14	.111E -13	.119E -13	.111E -13	.934E -14
450	.208E -13	.265E -13	.371E -13	.393E -13	.364E -13	.305E -13
350	.603E -12	.744E -12	.980E -12	.996E -12	.898E -12	.736E -12
250	.214E -10	.256E -10	.313E -10	.304E -10	.266E -10	.212E -10
150	.833E -09	.955E -09	.108E -08	.100E -08	.846E -09	.656E -09
100	.618E -08	.693E -08	.750E -08	.673E -08	.557E -08	.424E -08
50	.548E -07	.597E -07	.610E -07	.527E -07	.425E -07	.317E -07
0	.547E -06	.574E -06	.544E -06	.448E -06	.350E -06	.253E -06
-25	.218E -05	.223E -05	.200E -05	.160E -05	.122E -05	.865E -06
-50	.101E -04	.100E -04	.844E -05	.645E -05	.478E -05	.332E -05
-75	.360E -04	.347E -04	.277E -04	.205E -04	.148E -04	.101E -04

Table 3.1. Spectral absorption coefficient $\kappa_{ia}(\lambda)$ due to (1a) and (1b) reactions together as a function of λ and height h for the photospheric model of Maltby et al. (1986)

h[km]	$\kappa_{ia}(\lambda)$; λ [nm], κ_{ia}[cm^{-1}]					
	365	400	500	600	700	820
2040	.386E -17	.381E -17	.372E -17	.367E -17	.364E -17	.360E -17
1948	.916E -17	.897E -17	.861E -17	.840E -17	.826E -17	.813E -17
1810	.218E -16	.212E -16	.201E -16	.194E -16	.190E -16	.186E -16
1634	.670E -16	.648E -16	.608E -16	.583E -16	.567E -16	.553E -16
1406	.393E -15	.379E -15	.352E -15	.336E -15	.325E -15	.316E -15
1198	.225E -14	.217E -14	.200E -14	.190E -14	.183E -14	.178E -14
1079	.660E -14	.633E -14	.580E -14	.548E -14	.527E -14	.510E -14
912	.286E -13	.272E -13	.244E -13	.228E -13	.218E -13	.209E -13
862	.450E -13	.427E -13	.381E -13	.354E -13	.336E -13	.322E -13
761	.999E -13	.934E -13	.811E -13	.739E -13	.692E -13	.655E -13
711	.958E -13	.887E -13	.753E -13	.676E -13	.627E -13	.587E -13
655	.505E -13	.461E -13	.381E -13	.336E -13	.307E -13	.284E -13
605	.269E -13	.244E -13	.197E -13	.171E -13	.155E -13	.142E -13
553	.177E -13	.160E -13	.127E -13	.109E -13	.977E -14	.889E -14
528	.215E -13	.193E -13	.154E -13	.132E -13	.118E -13	.107E -13
503	.279E -13	.251E -13	.200E -13	.171E -13	.153E -13	.139E -13
478	.388E -13	.349E -13	.278E -13	.238E -13	.214E -13	.195E -13
453	.715E -13	.644E -13	.516E -13	.444E -13	.400E -13	.365E -13
428	.129E -12	.117E -12	.941E -13	.813E -13	.734E -13	.672E -13
403	.231E -12	.210E -12	.170E -12	.147E -12	.133E -12	.122E -12
378	.407E -12	.370E -12	.301E -12	.263E -12	.238E -12	.219E -12
352	.717E -12	.653E -12	.535E -12	.468E -12	.426E -12	.392E -12
301	.229E -11	.210E -11	.174E -11	.153E -11	.140E -11	.130E -11
250	.691E -11	.635E -11	.532E -11	.473E -11	.435E -11	.405E -11
200	.198E -10	.183E -10	.155E -10	.139E -10	.129E -10	.120E -10
175	.355E -10	.329E -10	.280E -10	.252E -10	.234E -10	.219E -10
150	.688E -10	.640E -10	.550E -10	.497E -10	.463E -10	.436E -10
125	.149E -09	.139E -09	.121E -09	.110E -09	.103E -09	.975E -10
100	.334E -09	.314E -09	.275E -09	.252E -09	.238E -09	.226E -09
75	.781E -09	.738E -09	.655E -09	.607E -09	.575E -09	.549E -09
50	.188E -08	.178E -08	.161E -08	.150E -08	.144E -08	.138E -08
35	.352E -08	.336E -08	.307E -08	.289E -08	.278E -08	.268E -08
20	.619E -08	.596E -08	.551E -08	.523E -08	.506E -08	.491E -08
10	.915E -08	.884E -08	.824E -08	.788E -08	.765E -08	.745E -08
0	.135E -07	.131E -07	.123E -07	.119E -07	.116E -07	.113E -07
-10	.199E -07	.194E -07	.184E -07	.179E -07	.175E -07	.172E -07
-20	.307E -07	.301E -07	.290E -07	.283E -07	.278E -07	.274E -07
-30	.478E -07	.471E -07	.459E -07	.452E -07	.447E -07	.443E -07
-40	.720E -07	.714E -07	.703E -07	.698E -07	.694E -07	.690E -07
-50	.105E -06	.104E -06	.104E -06	.104E -06	.104E -06	.103E -06
-60	.149E -06	.149E -06	.150E -06	.150E -06	.151E -06	.151E -06
-70	.205E -06	.206E -06	.209E -06	.211E -06	.212E -06	.213E -06
-80	.273E -06	.276E -06	.282E -06	.286E -06	.289E -06	.291E -06
-90	.348E -06	.352E -06	.362E -06	.369E -06	.374E -06	.377E -06
-100	.430E -06	.437E -06	.451E -06	.462E -06	.469E -06	.474E -06

Table 3.2. Same as in Table 3.1. but for the chromospheric model of Vernazza et al. (1981, model C)

h[km]	$\kappa_{ia}(\lambda)$; λ [nm], κ_{ia}[cm^{-1}]					
	365	400	500	600	700	820
2016	.274E -17	.270E -17	.264E -17	.260E -17	.258E -17	.256E -17
1990	.367E -17	.361E -17	.350E -17	.344E -17	.339E -17	.335E -17
1925	.628E -17	.615E -17	.591E -17	.576E -17	.567E -17	.558E -17
1785	.182E -16	.177E -16	.167E -16	.162E -16	.158E -16	.155E -16
1605	.673E -16	.652E -16	.611E -16	.586E -16	.570E -16	.556E -16
1515	.127E -15	.122E -15	.114E -15	.109E -15	.106E -15	.103E -15
1380	.346E -15	.333E -15	.310E -15	.296E -15	.286E -15	.278E -15
1280	.656E -15	.632E -15	.585E -15	.557E -15	.539E -15	.523E -15
1180	.138E -14	.133E -14	.123E -14	.116E -14	.112E -14	.109E -14
1065	.366E -14	.351E -14	.321E -14	.304E -14	.292E -14	.283E -14
980	.793E -14	.758E -14	.689E -14	.648E -14	.622E -14	.599E -14
905	.153E -13	.146E -13	.131E -13	.122E -13	.117E -13	.112E -13
855	.241E -13	.228E -13	.204E -13	.189E -13	.180E -13	.172E -13
755	.564E -13	.528E -13	.458E -13	.417E -13	.391E -13	.370E -13
705	.902E -13	.835E -13	.709E -13	.637E -13	.590E -13	.553E -13
655	.358E -13	.327E -13	.270E -13	.237E -13	.217E -13	.200E -13
605	.832E -14	.748E -14	.596E -14	.512E -14	.459E -14	.418E -14
555	.345E -14	.307E -14	.239E -14	.202E -14	.179E -14	.161E -14
515	.334E -14	.296E -14	.228E -14	.192E -14	.169E -14	.152E -14
450	.102E -13	.903E -14	.702E -14	.592E -14	.524E -14	.472E -14
350	.176E -12	.159E -12	.127E -12	.110E -12	.988E -13	.902E -13
250	.351E -11	.321E -11	.266E -11	.235E -11	.215E -11	.200E -11
150	.721E -10	.671E -10	.578E -10	.523E -10	.489E -10	.460E -10
100	.364E -09	.343E -09	.302E -09	.278E -09	.262E -09	.249E -09
50	.201E -08	.191E -08	.173E -08	.162E -08	.155E -08	.149E -08
0	.109E -07	.105E -07	.984E -08	.944E -08	.918E -08	.895E -08
-25	.280E -07	.274E -07	.263E -07	.256E -07	.252E -07	.248E -07
-50	.766E -07	.760E -07	.749E -07	.743E -07	.740E -07	.736E -07
-75	.175E -06	.175E -06	.176E -06	.178E -06	.178E -06	.179E -06

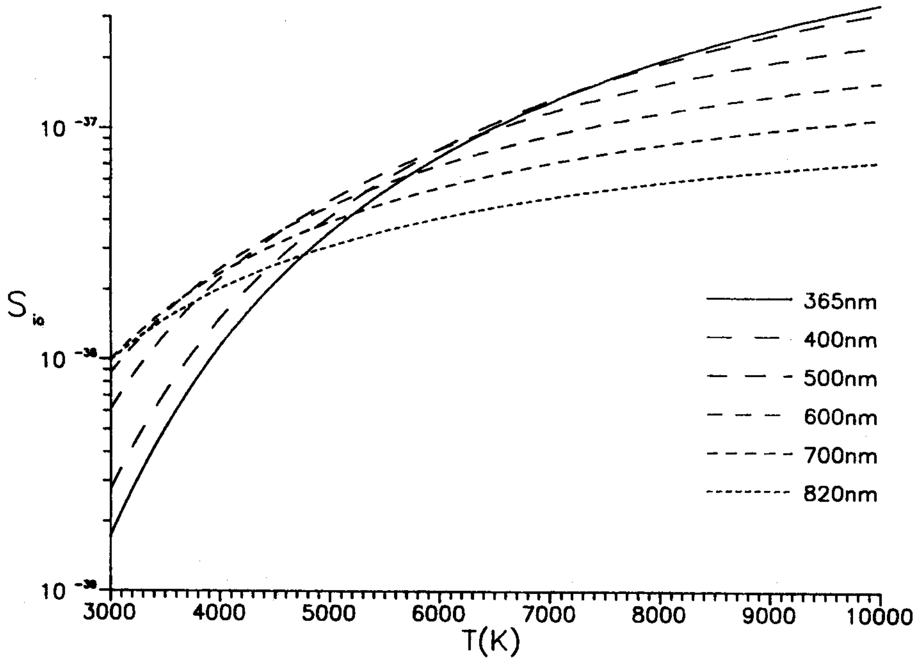

Fig. 1. Spectral coefficients $S_{\mathrm{ia}}(\lambda, T)$ for spontaneous emission due to (1a) and (1b) reactions together, are presented as a function of T for 365 nm $\leq \lambda \leq$ 820 nm

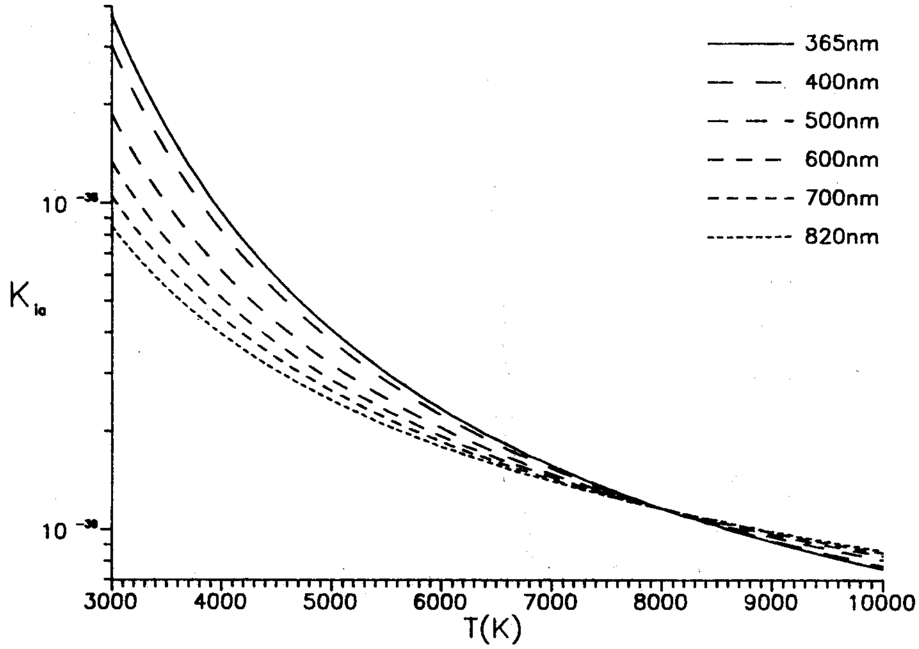

Fig. 2. Spectral coefficients $K_{\mathrm{ia}}(\lambda, T)$ for absorption due to (1a) and (1b) reactions together are presented as a function of T for 365 nm $\leq \lambda \leq$ 820 nm

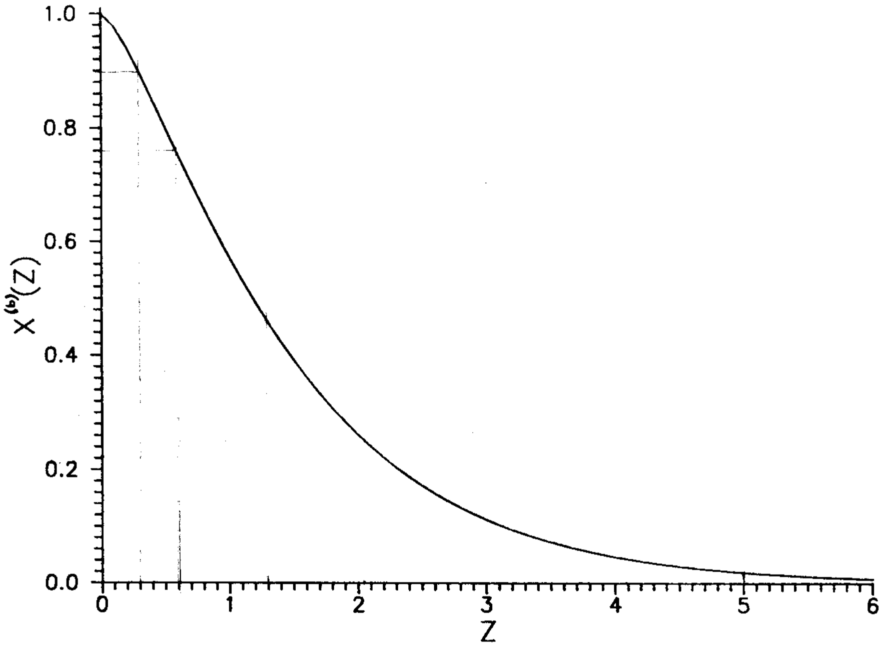

Fig. 3. Parameter $X^{(\mathrm{b})}(Z)$ defined by Eq. (11), for $Z \geq 0$

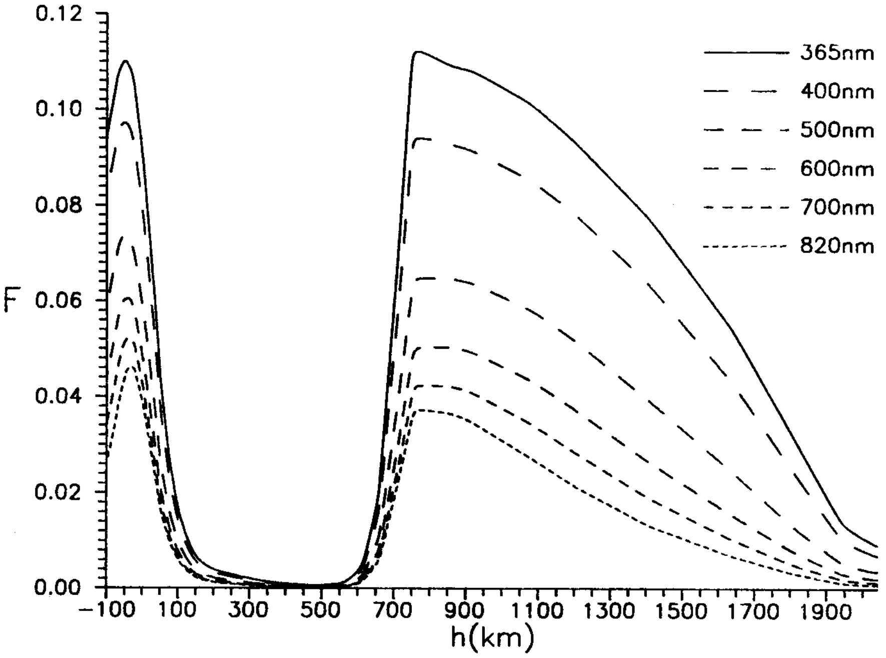

Fig. 4.1. Behavior of parameter $F(\lambda)$, defined by Eqs. (16) and (17), as a function of height h, for 365 nm $\leq \lambda \leq$ 820 nm for the photospheric model of Maltby et al. (1986)

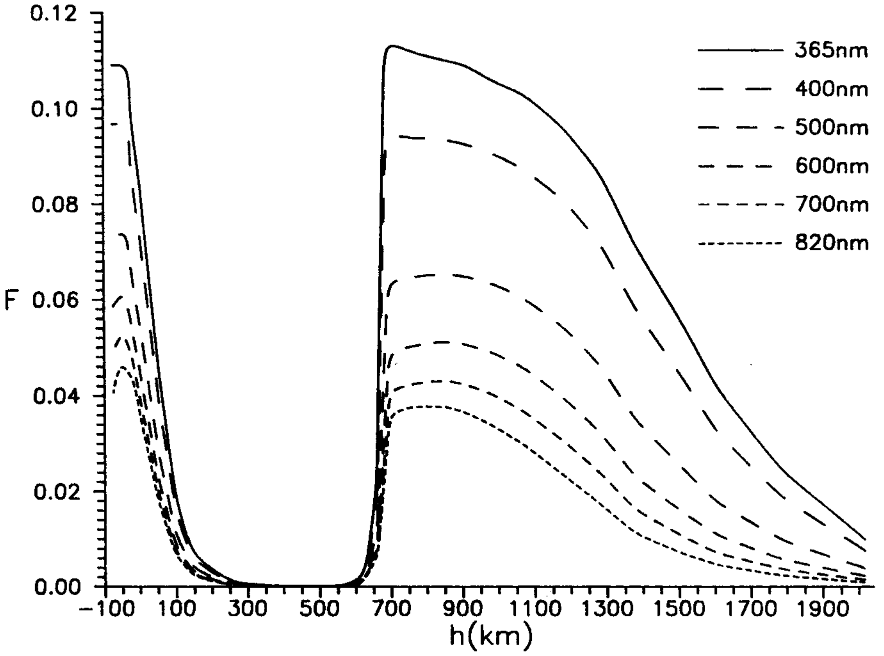

Fig. 4.2. Same as in Fig. 4.1. but for the chromospheric model of Vernazza et al. (1981, model C)

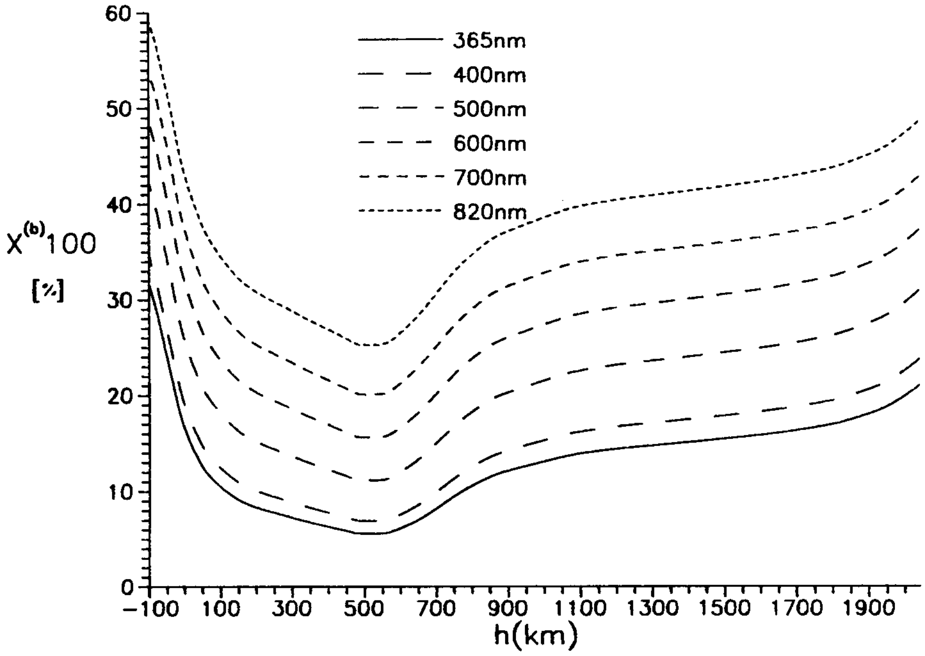

Fig. 5.1. Behavior of parameter $X^{(\mathrm{b})}(Z)$, where $Z = |U_1(R_\lambda)|/kT$, defined by Eqs. (9-14), as a function of height h, for 365 nm $\leq \lambda \leq$ 820 nm for the photospheric model of Maltby et al. (1986)

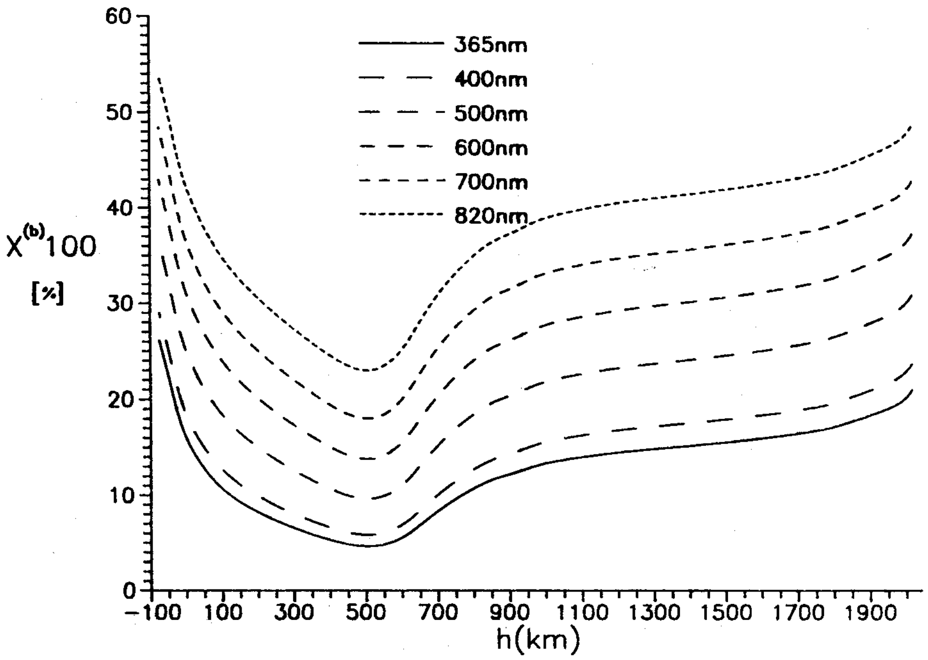

Fig. 5.2. Same as in Fig. 5.1. but for the chromospheric model of Vernazza et al. (1981, model C)