The contribution of ion-atom radiative collisions to the opacity of the solar atmosphere

A.A. Mihajlov^{1, 2}, M.S. Dimitrijević², and Lj.M. Ignjatović¹

¹ Institute of Physics, P.O. Box 57, 11001 Beograd, Yugoslavia

Received June 16, 1992; accepted February 3, 1993

Abstract. We investigate the contribution to the opacity of the solar atmosphere of some processes other than formation and photodissociation of the H⁻ ion. We show that positive-ion-atom radiative collision processes are not negligible at certain layers of the photosphere and of the chromosphere, although they make only a negligible contribution to the solar continuous optical emission emergent intensity.

Key words: atomic processes-molecular processes-Sun: photosphere-Sun: chromosphere

1. Introduction

The continuum opacity of the solar atmosphere in the visible range is dominated by radiation exchange in formation and dissociation of the H⁻ ion (see Mihalas 1978). However other processes involving atoms and positive ions may also play a role in the opacity. The purpose of the present paper is to evaluate the contribution of some ion-atom radiative collision processes to the opacity of the solar atmosphere. We will limit ourselves to the optical part of the electromagnetic spectrum (350-1250 nm) where the theory of the processes involved can be considered as correct.

Section 2 describes the processes involved; Sect. 3 contains a calculation of the corresponding emissivities, and Sect. 4 shows an application to the solar atmosphere. Sect. 5 is the conclusion.

2. Atomic and molecular radiative processes in the solar atmosphere

The following positive-ion-atom radiative collision processes will be treated here:

$$A^+ + A \leftrightarrow \varepsilon_{\lambda} + \begin{cases} A + A^+ & \text{(radiative charge exchange),} \\ A_2^+ & \text{(ion-atom photoassociation and photodissociation of molecular ion),} \end{cases}$$

Send offprint requests to: M.S. Dimitrijević

where all atomic particles are in their electronic ground states and $\varepsilon_{\lambda} = 2\pi\hbar c/\lambda$ is the energy of the photon with the wavelength λ .

The contribution of processes (1) to the opacity of the solar atmosphere will be compared here with the contribution of the following known radiative processes:

(i) free-free and free-bound process involving a positive ion:

$$A^{+} + e \leftrightarrow \varepsilon_{\lambda} + \begin{cases} A^{+} + e \\ A^{*}, \end{cases} \tag{2}$$

where A* is an atom in an excited state:

(ii) free-free processes in the field of an atom:

$$A + e \leftrightarrow \varepsilon_{\lambda} + A + e; \tag{3}$$

(iii) free-bound processes in the field of an atom (first of all formation and dissociation of H^-):

$$A + e \leftrightarrow \varepsilon_{\lambda} + A^{-}. \tag{4}$$

The theory of processes (1) has been given by Drukarev & Mihajlov (1974), Mihajlov & Popović (1981), Mihajlov & Dimitrijević (1986). The molecular ion (H^{*}₂) photodissociation have been described by e.g. Mihalas (1978). Process (2) has been treated by e.g. Menzel (1962) and Sobelman (1979). Process (3) has been studied by e.g. Firsov & Chibisov (1960) and Stilley & Callaway (1970), while process (4) has been investigated by e.g. Armstrong (1963), Welsner & Armstrong (1964), and Wishart (1979). Processes (3) and (4) have been described as well e.g. in Mihalas (1978).

The spectral emissivity corresponding to process (1) is:

$$\varepsilon_{ia}(\lambda) = \sum S_{ia}^{A}(\lambda) \ N(A^{+}) \ N(A), \tag{5a}$$

where $N(A^+)$ and N(A) are the number density of the ion A^+ and the atom A respectively, the sum is over the different kinds of ions and atoms and $S_{ia}^A(\lambda)$ is the emission spectral coefficient characterizing the total contribution of both photoemissive channels. It is given in analytical form by Mihajlov & Popović (1981).

Similarly we have for process (2):

$$\varepsilon_{\rm ei}(\lambda) = \sum S_{\rm ei}^{\rm A}(\lambda) N({\rm A}^{+}) N_{\rm e}, \tag{5b}$$

² Astronomical Observatory, Volgina 7, 11050 Beograd, Yugoslavia

where N_e is the electronic density and $S_{ei}^{A}(\lambda)$ is given analytically by Sobelman (1979).

For processes (3) and (4) we have respectively:

$$\varepsilon_{\text{ea}}^{\text{ff}}(\lambda) = \sum S_{\text{ea}}^{\text{A,ff}}(\lambda) \ N(\text{A}) N_{\text{e}}, \tag{5c}$$

$$\varepsilon_{\rm ea}^{\rm fb}(\lambda) = \sum S_{\rm ea}^{\rm A,fb}(\lambda) \ N({\rm A}) N_{\rm e}. \tag{5d}$$

We also define the ratios of emissivities:

$$F_{ei}(\lambda) = \varepsilon_{ia}(\lambda)/\varepsilon_{ei}(\lambda),$$
 (6a)

$$F_{ea}^{ff}(\lambda) = \varepsilon_{ia}(\lambda)/\varepsilon_{ea}^{ff}(\lambda),$$
 (6b)

$$F_{\rm ea}^{\rm fb}(\lambda) = \varepsilon_{\rm ia}(\lambda)/\varepsilon_{\rm ea}^{\rm fb}(\lambda),$$
 (6c)

characterizing relative contribution of the process (1) in comparison with processes (2), (3) and (4) respectively.

In the case of local thermal equilibrium, these ratios are also the ratios of the corresponding absorption coefficients. We now proceed to calculating the quantities F.

3. Calculation of the emissivities

Hydrogen dominates the opacity in practice. Consequently we can write for process (1):

$$\varepsilon_{ia}(\lambda) = \xi_{ia} S_{ia}^{H}(\lambda) N(H^{+}) N(H), \tag{7a}$$

where $N(H^*)$ is the density of protons, N(H) the density of hydrogen atoms in the 1s state. The coefficient ξ_{ia} takes into account the effect of real plasma composition differences from hydrogen case. Here $\xi_{ia} \cong 1$.

Other ions intervene in process (2) for which we can write:

$$\varepsilon_{ei}(\lambda) = \xi_{ei} S_{ei}^{H}(\lambda) N_{e} N_{i}, \tag{7b}$$

where $\xi_{ei} \approx 1$ is the Biberman-Norman factor and $N_i = \Sigma N(A^+)$ is the total density of single-charged positive ions.

Similarly we can write for processes (3) and (4):

$$\varepsilon_{ea}^{ff}(\lambda) = \xi_{ea}^{ff} S_{ea}^{H,ff}(\lambda) N_e N(H), \tag{7c}$$

$$\varepsilon_{ea}^{fb}(\lambda) = \xi_{ea}^{fb} S_{ea}^{H,fb}(\lambda) N_e N(H), \tag{7d}$$

where the coefficients $\xi_{ea}^{ff,fb} \cong 1$.

The expression of $S_{ia}^{H}(\lambda, T)$ is (Mihajlov & Popović 1981):

$$S_{ia}^{H}(\lambda, T) = 4.7769 \, 10^{-34} \frac{C(R_{\lambda})(R_{\lambda}/a_{o})^{4}}{1 - a_{o}/R_{\lambda}} \left(\frac{\varepsilon_{\lambda}}{2Ry}\right)^{5}$$

$$\times \exp\left[-\frac{U_2(R_{\lambda})}{kT}\right],$$

$$C(R_{\lambda}) = \left\lceil \frac{2D_{12}(R_{\lambda})}{eR_{\lambda}} \right\rceil^{2} \frac{1 - a_{o}/R_{\lambda}}{\gamma(R_{\lambda})}, \tag{8}$$

where R is the distance between protons (in the system $H^+ + H$),

$$\gamma(R_{\lambda}) = \left| \frac{\mathrm{d} \ln[E_{12}(R)/2Ry]}{\mathrm{d}(R/a_{\mathrm{o}})} \right|_{R=R_{\lambda}},$$

$$E_{12}(R) = U_2(R) - U_1(R), (9)$$

 $U_1(R)$ and $U_2(R)$ —adiabatic terms of the ground $(1\Sigma_g)$ and the first excited $(1\Sigma_u)$ electronic states of ion H_2^+ and $D_{12}(R)$ —modulus of dipole matrix element (between these states). Finally, R_2 is the root of the equation:

$$E_{12}(R) = \varepsilon_{\lambda}. \tag{10}$$

In Table 1 values of R_{λ} as well as of other parameters needed for the determination of $S_{ia}^{H}(\lambda, T)$ in 250 nm $\leq \lambda \leq 1500$ nm range are presented in atomic units. These parameters have been determined using $U_{1}(R)$ and $U_{2}(R)$ values from Bates et al. (1953) and $D_{12}(R)$ values from Ramaker & Peek (1973).

The spectral coefficient $S_{ei}^{H}(\lambda, T, N_e)$ is evaluated in the quasi-classical approximation (Sobelman 1979) as:

$$S_{ei}^{H}(\lambda, T; N_{e}) = 1.753 \cdot 10^{-33} \left(\frac{2Ry}{kT}\right)^{1/2} \left(\frac{\varepsilon_{\lambda}}{2Ry}\right)^{2} \exp\left[-\frac{\varepsilon_{\lambda}}{kT}\right] \times \left\{1 + \frac{2Ry}{KT} \sum_{n=0}^{N_{max}} \frac{1}{n^{3}} \exp\left[\frac{Ry}{n^{2}kT}\right]\right\}, \quad (11)$$

where

$$\lambda_1 = \frac{2\pi c\hbar}{Ry}, \quad n_{\text{max}} = \left(\frac{e^2}{r_D Ry}\right)^{-1/2},$$
 (12)

n-principal quantum number of excited hydrogen state and $r_D = r_D(T; N_e)$ -Debye radius.

With numerical coefficients given in Eqs. (8) and (11), corresponding spectral coefficients $S_{ia}^{H}(\lambda, T)$ and $S_{ei}^{H}(\lambda, T; N_e)$ are expressed in [cm³ J s⁻¹ nm⁻¹] units.

For $\varepsilon_{\rm ea}^{\rm ff}(\lambda)$ we will use the tabulated data of Stilley & Callaway (1970), while the data for $\varepsilon_{\rm ea}^{\rm fb}(\lambda)$ are from tables of Wishart (1979).

4. Result for the solar atmosphere and discussion

We now apply the previous calculation to the solar atmosphere using the solar photospheric model of Maltby et al.

Table 1. Parameters of molecular ion H_2^+ in a.u., needed for the $S_{ia}^H(\lambda)$ calculation, by 250 nm $\leq \lambda \leq 1500$ nm

λ	R_{λ}	$C(R_{\lambda})$	$E_{12}(R_{\lambda})$	$U_1(R_{\lambda})$	$U_2(R_{\lambda})$
(nm)	(a.u.)	(a.u.)	(a.u.)	(a.u.)	(a.u.)
250	3.17	0.875	0.1820	0.0684	0.1140
300	3.42	0.901	0.1520	0.0605	0.0913
350	3.63	0.922	0.1300	0.0544	0.0758
400	3.81	0.939	0.1140	0.0494	0.0645
450	3.98	0.954	0.1010	0.0453	0.0559
500	4.12	0.966	0.0911	0.0419	0.0492
600	4.37	0.987	0.0759	0.0364	0.0395
700	4.58	1.000	0.0651	0.0323	0.0328
800	4.77	1.020	0.0569	0.0291	0.0279
900	4.93	1.030	0.0506	0.0264	0.0242
1000	5.05	0.993	0.0456	0.0238	0.0217
1250	5.34	0.984	0.0364	0.0196	0.0169
1500	5.57	0.981	0.0304	0.0166	0.0137

(1986) (their Table 11, photospheric reference model) for altitudes (h) lower than 605 km and chromospheric model of Vernazza et al. (1981) (their model C) for higher altitudes.

Taking into account Eqs. (5, 6, 7) we will represent quantities F as:

$$F_{\mathrm{ei}}(\lambda) = f_{\mathrm{ei}} \frac{S_{\mathrm{ia}}^{\mathrm{H}}(\lambda)}{S_{\mathrm{ei}}^{\mathrm{H}}(\lambda)} \eta_{\mathrm{ei}},$$

$$F_{\text{ea}}^{\text{ff}}(\lambda) = f_{\text{ea}}^{\text{ff}} \frac{S_{\text{ia}}^{\text{H}}(\lambda)}{S_{\text{ea}}^{\text{H,ff}}(\lambda)} \eta_{\text{ea}},$$

$$F_{\rm ea}^{\rm fb}(\lambda) = f_{\rm ea}^{\rm fb} \frac{S_{\rm ia}^{\rm H}(\lambda)}{S_{\rm ea}^{\rm H,fb}(\lambda)} \eta_{\rm ea},$$

$$\eta_{ei} = \frac{N(H^+)N(H)}{N_eN_i}, \qquad \eta_{ea} = \frac{N(H^+)}{N_e},$$

where correction factors

$$f_{\rm ei} = \frac{\xi_{\rm ia}}{\xi_{\rm ei}}, \quad f_{\rm ea}^{\rm ff,fb} = \frac{\xi_{\rm ia}}{\xi_{\rm ea}^{\rm ff,fb}},$$

are close to unity since in our case quantities $\xi \cong 1$. The factor η_{ei} and η_{ea} values are presented in Table 2 together with basic parameters of the used solar models in the range $-100 \text{ km} \le h \le 1605 \text{ km}$.

The comparison of (1) and (2) processes contribution is presented in Figs. 1a, b where the behavior of $F_{ia}(\lambda)$ is shown. We can see two maxima, one in the photosphere and the other one in the chromosphere. This fact is explained by two corresponding strong maxima of the η_{ei} factor shown in Table 2. In consequence, although $S_{ia}^H/S_{ei}^H \ll 1$, $F_{ei}(\lambda)$ becomes larger than 1 at those locations, as illustrated in Figs. 1a and 1b.

Table 2. Values of η_{ei} and $\dot{\eta}_{ea}$ in the h-range considered. Up to h=605 km the model of photosphera (Maltby 1986) and for h>605 km the model of chromosphere (Vernazza et al. 1981) have been used

h (km)	<i>T</i> (k)	N _e (cm ⁻³)	N _H (cm ⁻³)	η_{ei}	η_{ca}
1605	6440	0.601 1011	0.639 1012	0.963 10 ¹	0.100 10 ¹
1515	6370	0.64610^{11}	0.10510^{13}	0.15210^2	0.10010^{1}
1380	6280	0.76010^{11}	0.22710^{13}	0.28910^2	0.10010^{1}
1280	6220	0.74910^{11}	0.42010^{13}	$0.551\ 10^{2}$	0.10010^{1}
1180	6150	0.81110^{11}	0.78710^{13}	0.96010^2	0.10010^{1}
1065	6040	0.93510^{11}	0.17110^{14}	0.18210^3	0.10010^{1}
980	5925	0.10410^{12}	0.31510^{14}	0.301 10 ³	0.10010^{1}
855	5650	0.10610^{12}	0.81410^{14}	0.76410^3	0.10010^{1}
755	5280	0.88410^{11}	0.18610^{15}	0.211 104	0.10010^{1}
705	5030	0.76610^{11}	0.294 1015	0.383 104	0.10010^{1}
655	4730	0.80910^{11}	0.47910^{15}	0.111104	0.187
605	4420	0.11110^{12}	0.81210^{15}	0.10610^3	0.14510^{-1}
553	4410	0.19810^{12}	0.164 1016	0.71110 ²	0.85610^{-2}
478	4410	0.37910^{12}	0.33710^{16}	0.42210^2	0.47610^{-2}
428	4510	0.58910^{12}	0.52710^{16}	0.63510^2	0.708 10-2
378	4610	0.91310^{12}	0.81710^{16}	0.901 10 ²	0.10110^{-1}
301	4770	0.17310^{13}	0.15610^{17}	0.15910^3	0.17710^{-1}
200	4990	0.40010^{13}	0.35010^{17}	0.30210^3	0.34410^{-1}
150	5150	0.61410^{13}	0.50910^{17}	0.491 10 ³	0.59310^{-1}
100	5410	0.10010^{14}	0.71410^{17}	0.105 104	0.147
50	5790	0.19810^{14}	0.96010^{17}	0.18510^4	0.381
20	6180	0.405 1014	0.11010^{18}	0.175 104	0.642
0	6520	0.768 1014	0.11910^{18}	0.122 104	0.791
-20	6980	0.17310^{15}	0.12510^{18}	0.65210^3	0.901
-40	7590	0.44610^{15}	0.12910^{18}	0.27710^3	0.966
-60	8220	0.10510^{16}	0.13110^{18}	0.12410^3	0.994
-80	8860	0.22110^{16}	0.13310^{18}	0.59210^2	0.10010^{1}
-100	9400	0.38610^{16}	0.13610^{18}	0342 10 ²	0.10010^{1}

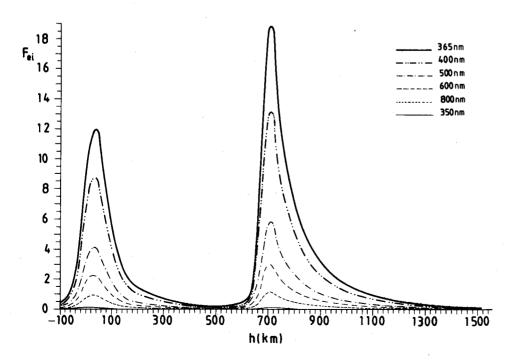


Fig. 1a. The parameter $F_{\rm ei}(\lambda)$, for $f_{\rm ei}=1$, in the 350 nm $\leq \lambda \leq$ 800 nm range (350 nm $< \lambda_2 <$ 365 nm), as a function of h

Figures 1a, b represent as well the behavior of $F_{ei}(\lambda)$ around $\lambda = \lambda_n$ for n = 2 and 3. Here, $\lambda_n = n^2 \lambda_1$ are continuum limits caused by free-bound electron transitions in n-states, and λ_1 is given by Eq. (12). The $\lambda = \lambda_n$ points are the discontinuity points of $S_{ei}^H(\lambda)$. In these points $S_{ei}^H(\lambda)$ has a jump caused by exclusion (when λ increases) or inclusion (when λ decreases) of the corresponding continuum. The described situation is well illustrated in Figs. 1a and 1b.

We can see that within the height range considered the contribution of processes (2) to the opacity may be neglected. However outside of this region its relative contribution to the processes (1) increases sharply.

The comparison of the contribution of processes (1) and (3) is presented in Fig. 2 where the behavior of $F_{\rm ea}^{\rm ff}(\lambda)$ is shown. One can see the photospheric and the chromospheric maximum of $F_{\rm ea}^{\rm ff}(\lambda)$ located in the same height range as for $F_{\rm ei}(\lambda)$ in Figs. 1a, b. These maxima however are of smaller intensity.

In the considered photospheric and chromospheric regions, electron-atom processes are always dominant for wavelengths larger than 1250 nm, including the infrared part of the spectrum at 1650 nm, where the minimum in ${\rm H^-}$ absorption occurs. Our calculation however, show that for sophisticated investigations ion-atom processes

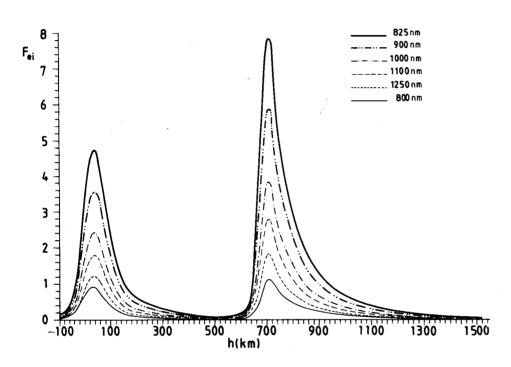


Fig. 1b. The parameter $F_{ei}(\lambda)$, for $f_{ei} = 1$, in the 800 nm $\leq \lambda \leq 1250$ nm range (800 nm $< \lambda_3 < 850$ nm), as a function of h

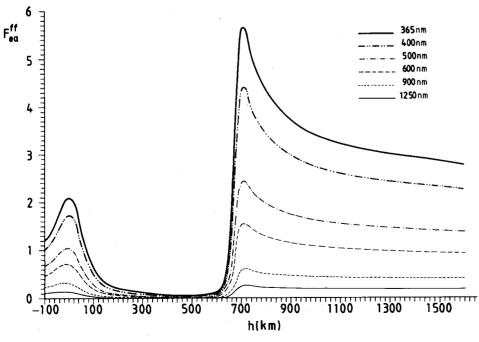


Fig. 2. The parameter $F_{ea}^{ff}(\lambda)$, for $f_{ea}^{ff}=1$, in the $350 \text{ nm} \le \lambda \le 1250 \text{ nm}$ range, as a function of h

must be taken into account around 1650 nm since in the layers considered, the contribution of these processes is 5-15% of that of electron-atom ones.

The comparison of the contributions of processes (1) and (4) is presented in Figs. 3a, b where the behavior of $F_{\rm ea}^{\rm fb}(\lambda)$ is shown. One can see in these figures that the behavior of $F_{\rm ea}^{\rm fb}(\lambda)$ is not a monotonous function of λ , which is the consequence of the behaviour of the electron radiative capture cross section or the ion H⁻ photodissoci-

ation cross section. These figures demonstrate that the values of $F_{\rm ea}^{\rm fb}(\lambda)$ parameter change from 0.15 up to 0.05 in the $-100~{\rm km}~{\leq}\,h{\leq}\,50~{\rm km}$, decrease slowly up to around 700 km and increases steeply up to 0.1. After 700 km they increase very slowly for all λ considered.

In Figs. (1)-(3) we can see that the processes (1), not taken into account up to now in photosphere and chromosphere research from the spectroscopical point of view are not negligible and that in particular layers it becomes in

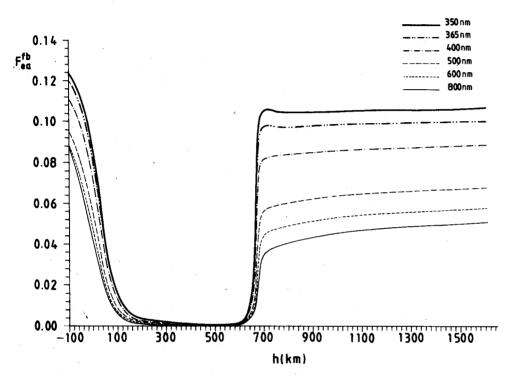


Fig. 3a. The parameter $F_{\rm ea}^{\rm fb}(\lambda)$, for $f_{\rm ea}^{\rm fb}=1$, in the 350 nm $\leq \lambda \leq 800$ nm range, as a function of h

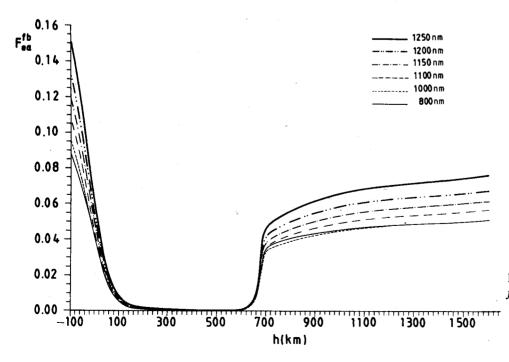


Fig. 3b. The parameter $F_{\rm ea}^{\rm fb}(\lambda)$, for $f_{\rm ea}^{\rm fb}=1$, in the 800 nm $\leq \lambda \leq$ 1250 nm range, as a function of h

fact even comparable with processes (4), the most important for continuous emission (absorption) spectrum formation for the height range considered.

The importance of including H_2^+ in continuum opacity calculations is well known for A type star atmospheres (see e.g. Mihalas 1978), where hydrogen is approximately half ionized. In this case the H_2^+ photodissociation contribution to the total absorption is around 10%. We want to draw attention that on the basis of Fig. 3 one can see that the inclusion of radiative charge exchange processes together with H_2^+ photodissociation results in a combined contribution to the total absorption spectrum of around 10% for particular photospheric and chromospheric layers of the Sun, in spite of the fact that only 10^{-2} – 10^{-4} of hydrogen is ionized.

Besides hydrogen, there is also helium but its density is below 10% of that of hyrogen in the regions considered (Vernazza et al. 1981). Consequently, taking into account the large ionization potential of helium, electron ion-radiative processes involving helium may be neglected in present case. We can neglect as well emission (absorption) due to electron scattering on He in comparison with emission (absorption) due to electron scattering on H. Our calculation on the basis of the paper by Bell et al. (1982) shows that the contribution of the e-He radiative process is always below 3% of that of e-H in the regions considered. Moreover, the He⁻ ion in the ground state is not stable and the He ion stable states are created from the excited helium atom and have thus a high creation threshold (around 20 eV, see e.g. Massey 1976). Consequently, radiative processes with He⁻ participation may be neglected in our case as well.

We have estimated (for hydrogen) the relative importance of processes (1) on the total optical depth of solar atmosphere in the range $0 \text{ km} \le h \le 1065 \text{ km}$, as well as on the emergent intensity (see e.g. Mihalas 1978). The calculations have been performed for radial rays. This shows that the contribution of processes (1) changes the optical depth of the atmospheric layer considered (photosphere and lower chromosphere) from 3.5 up to 1.5% when λ varies from 400 up to 800 nm.

Such an optical depth change is not negligible for sophisticated calculations. However it does not influence significantly the emergent intensity. Our calcultions show that for the photosphere (0 km $\leq h \leq$ 605 km). the change of emergent intensity is between 0.28 to 0.14% for λ between 500 and 800 nm. In this layer for $\lambda \geq$ 500 nm, the

source function may be approximated by a Planck function, as used here.

5. Conclusions

The present results show that some atomic processes generally neglected can contribute appreciably to the opacity of some layers of the solar photosphere and chromosphere, and should not be neglected for local studies. So, the contribution of positive-ion-atom radiative collisions changes the optical depth of photosphere and lower chromosphere in the spectral range considered by 3.5-1.5%. However it changes only by about 0.2% of the emergent emission intensity from the solar photosphere in the 500-800 nm spectral range and can be neglected in this case. The processes we have studied may also be of importance in other stars where the temperature of the atmosphere is smaller than 10⁴ K. Some other processes involving other elements than hydrogen or collisions between H⁻ and H might also be of interest.

Acknowledgements. We are indebted to Professor P. Maltby for very illuminating discussion. We are also grateful to Professor James Lequeux for his help to improve our article.

References

Armstrong B.H., 1963, Phys. Rev. 131, 1132

Bates D.R., Ledsham K., Stewart A.L., 1953, Phil. Trans. R. Soc. Lond., Ser. A 246, 215

Bell K.L., Berrington K.A., Croskery J.P., 1982, J. Phys. B 15, 977 Drukarev G.P., Mihajlov A.A., 1974, Opt. Spectrosk. 37, 384 Firsov O.B., Chibisov M.I., 1960, Zh. Ehksp. Teor. Fiz. 39, 1770

Maltby P., Avrett E.H., Carlsson M., Kjeldseth-Moe O., Kurucz R.L., Loeser R., 1986, ApJ 306, 284

Massey H.S.W., 1976, Negative Ions. Cambridge University Press, Cambridge

Menzel D.H., 1962, Selected Papers on Physical Processes in Ionized Plasmas. Dover Publications, New York

Mihalas D., 1978, Stellar Atmospheres, Chap. 7.7. W.H. Freeman, San Francisco

Mihajlov A.A., Popović M.M., 1981, Phys. Rev. A 23, 1679

Mihajlov A.A., Dimitrijević M.S., 1986, A&A 155, 319

Ramaker D.E., Peek J.M., 1973, Atomic Data 5, 167

Sobelman I.I., 1979, Atomic Spectra and Radiative Transitions, Chap. 9.5. Springer, Berlin

Stilley J.L., Callaway J., 1970, ApJ 160, 245

Vernazza J.E., Avrett E.H., Loeser R., 1981, ApJS 45, 635

Weisner J.D., Armstrong B.H., 1964, Proc. Phys. Soc. 83, 31

Wishart A.W., 1979, MNRAS 187, 59